Ultra Scientist Vol. 24(3)B, 517-528 (2012).

Thermal diffusion and Radiation effects on Unsteady MHD Free Convection Flow Past an Inclined plate with Variable temperature and Mass diffusion in the presence of Heat source/sink

A.G. VIJAYA KUMAR*, Y. RAJASHEKARA GOUD**,
K. NANDA GOPAL* and S.V.K. VARMA***

(Acceptance Date 30th November, 2012)

Abstract

An analytical study is performed is to investigate thermal diffusion and radiation effects on unsteady MHD flow past an inclined plate with variable temperature and mass diffusion in the presence of heat source or sink under the influence of applied transverse magnetic field. The fluid considered here is a gray, absorbing/ emitting radiation but a non-scattering medium. At time t>0, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane. And at the same time, the plate temperature and concentration levels near the plate raised linearly with time t. The dimensionless governing equations involved in the present analysis are solved using the Laplace transform technique. The velocity, temperature, concentration, the rate or heat transfer and the rate of mass transfer are studied through graphs and tables in terms of different physical parameters like magnetic field parameter (M), radiation parameter (R), heat source parameter (H), Schmidt parameter (Sc), inclination parameter (α), soret number (So), Prandtl number (Pr), thermal Grashof number (Gr), mass Grashof number (Gm) and time (t).

Key words: MHD, heat and mass transfer, thermal diffusion, exponentially, Accelerated, inclined plate, radiation.

Nomenclature :

- a^* Absorption coefficient
- a Accelerated parameter
- H Heat source parameter
- B_0 External magnetic field
- C' Species concentration
- C'_w Concentration of the plate
- C'_∞ Concentration of the fluid far away from the plate
- C Dimensionless concentration
- C_p Specific heat at constant pressure
- D Chemical molecular diffusivity
- D_1 Coefficient of thermal diffusivity
- g Acceleration due to gravity
- G_r Thermal Grashof number
- G_m Mass Grashof number
- M Magnetic field parameter
- Nu Nusselt number
- Pr Prandtl number
- q_r Radiative heat flux in the y- direction
- R Radiative parameter
- Sc Schmidt number
- So Soret number
- Sh Sherwood number
- T' Temperature of the fluid near the plate
- T'_w Temperature of the plate
- T'_∞ Temperature of the fluid far away from the plate
- t' Time
- t Dimensionless time
- u' Velocity of the fluid in the x'- direction
- u_0 Velocity of the plate
- u Dimensionless velocity
- y' Co-ordinate axis normal to the plate
- y Dimensionless co-ordinate axis normal to the plate

Greek symbols:

- κ Thermal conductivity of the fluid
- α Thermal diffusivity
- β Volumetric coefficient of thermal expansion
- β^* Volumetric coefficient of expansion with concentration
- μ Coefficient of viscosity
- υ Kinematic viscosity
- ρ Density of the fluid
- σ Electric conductivity
- θ Dimensionless temperature
- erf Error function
- erfc Complementary error function

Subscripts:

- ω Conditions on the wall
- ∞ Free stream conditions

Introduction

In nature, there exist flows which are caused not only by the temperature differences but also the concentration differences. These mass transfer differences do affect the rate of heat transfer. In industries, many transport processes exist in which heat and mass transfer takes place simultaneously as a result of combined buoyancy effect in the presence of thermal radiation. Hence, Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

The study of magneto hydro-dynamics with mass and heat transfer in the presence of radiation and diffusion has attracted the attention of a large number of scholars due to diverse applications. In astrophysics and geophysics, it is applied to study the stellar and solar structures, radio propagation through the ionosphere, etc. In engineering we find its applications like in MHD pumps, MHD bearings, *etc*. The phenomenon of mass transfer is also very common in theory of stellar structure and observable effects are detectable on the solar surface. In free convection flow the study of effects of magnetic field play a major rule in liquid metals, electrolytes and ionized gases. In power engineering, the thermal physics of hydro magnetic problems with mass transfer have enormous applications. Radiative flows are encountered in many industrial and environment processes, *e.g*. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. MHD effects on impulsively started vertical infinite plate with variable temperature in the presence of transverse magnetic field were studied by Soundalgekar *et al*.[12]. The effects of transversely applied magnetic field, on the flow of an electrically conducting fluid past an impulsively started infinite isothermal vertical plate were also studied by Soundalgekar *et al*.[11]. The dimensionless governing equations were solved using Laplace transform technique. Kumari and nath[8] studied the development of the asymmetric flow of a viscous electrically conducting fluid in the forward stagnation point region of a two-dimensional body and over a stretching surface was set into impulsive motion from the rest. The governing equations were solved using finite difference scheme. The radiative free convection flow of an optically thin gray-gas past semi-infinite vertical plate studied by Soundalgekar and Takhar[13]. Hossain and Takhar have considered radiation effects on mixed convection along an isothermal vertical plate[5]. In all above studies the stationary vertical plate considered. Raptis and Perdikis[10] studied the effects of thermal-radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al*.[4] have considered radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Muthucumaraswamy and Janakiraman[9] have studied MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion.

Alam and Sattar[3] have analyzed the thermal-diffusion effect on MHD free convection and mass transfer flow. Jha and Singh[6] have studied the importance of the effects of thermal-diffusion(mass diffusion due to temperature gradient). Alam *et al*.[1] studied the thermal-diffusion effect on unsteady MHD free convection and mass transfer flow past an impulsively started vertical porous plate. Recently, Alam *et al*.[2], studied combined free convection and mass transfer flow past a vertical plate with heat generation and thermal-diffusion through porous medium. Rajesh and Varma[14] studied thermal diffusion and radiation effects on MHD flow past a vertical plate with variable temperature and mass diffusion. Recently, Kumar and Varma[15] investigated thermal diffusion and radiation effects on unsteady MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature and variable mass diffusion.

The aim of the present work is to study the effects of thermal-diffusion and radiation on unsteady Hydro-magnetic flow past an inclined surface of an electrically conducting, radiating viscous, incompressible fluid in the presence of heat source and applied transverse magnetic field. The dimensionless governing equations involved in the present analysis are solved using Laplace transform technique. The solutions are expressed in terms of exponential and complementary error functions.

Mathematical formulation :

In this problem, we consider an unsteady hydro magnetic two-dimensional laminar free convection flow of a viscous, incompressible, electrically conducting, radiating fluid past an infinite inclined plate with an acute angle (α) to the vertical. The flow is assumed to be in x'-direction, which is taken along the infinite inclined plate and $y'-$ axis is taken normal to it. Initially it is assumed that the plate and fluid are at the same temperature T'_∞ and concentration level C'_∞ in stationary condition for all the points. At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in the vertical upward direction against to the gravitational field. And at the same time the plate temperature is raised linearly with time t. And also the mass is diffused from the plate to the fluid is linearly with time. A transverse magnetic field of uniform strength B_0 is assumed to be applied normal to the direction of flow. In this analysis, it is assumed that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. The fluid considered here is gray, absorbing/emitting radiation but a non-scattering medium. It is also assumed that all the fluid properties are constant except the influence of the density variation with temperature and concentration in the body force term. Then under by usual Boussinesq's approximation, the unsteady flow is governed by the following equations.

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty)\cos\alpha + g\beta^*(C' - C'_\infty)\cos\alpha + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma\beta_0^2 u'}{\rho} \tag{1}$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r}{\partial y'} + Q'(T'_\infty - T') \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} + D_1\left(\frac{\partial^2 T'}{\partial y'^2}\right) \tag{3}$$

With the following initial and boundary conditions

$$t' \le 0 : u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty, \ \textit{for all } y'$$

$$t' > 0 : \ u' = u_0 \exp(a't'), \ T' = T'_\infty + (T'_w - T'_\infty)At',$$
$$C' = C'_\infty + (C'_w - C'_\infty)At' \quad at \quad y' = 0$$

$$u' = 0, \ T' \to T'_\infty, \ C' \to C'_\infty \ \ as \ y' \to \infty \tag{4}$$

Where $A = \frac{u_0^2}{v}$. The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y'} = -4a^* \sigma \left(T_\infty'^4 - T'^4 \right) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small and that T'^4 may be expressed as a linear function of the temperature. This is obtained by expanding T'^4 in a Taylor series about T'_∞ and neglecting the higher order terms, thus we get

$$T'^4 \cong 4T_\infty'^3 T' - 3T_\infty'^4 \tag{6}$$

From equations (5) and (6), equation (2) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y'^2} + 16a^* \sigma T_\infty'^3 \left(T'_\infty - T' \right) \tag{7}$$

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{u_0}, t = \frac{t' u_0^2}{v}, y = \frac{y' u_0}{v}, \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, P_r = \frac{\mu C_\rho}{\kappa}, S_0 = \frac{D_1 (T'_w - T'_\infty)}{v (C'_w - C'_\infty)}$$

$$G_r = \frac{g\beta v \left(T'_w - T'_\infty \right)}{u_0^3}, G_m = \frac{g\beta^\bullet v \left(C'_w - C'_\infty \right)}{u_0^3}, \tag{8}$$

We get the following governing equations which are dimensionless

$$\frac{\partial u}{\partial t} = G_r \theta \cos\alpha + G_m C \cos\alpha + \frac{\partial^2 u}{\partial y^2} - Mu, \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr} \frac{\partial^2 \theta}{\partial y^2} - \frac{1}{\Pr} \left(R + H \right) \theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} + S_o \frac{\partial^2 \theta}{\partial y^2} \tag{11}$$

The initial and boundary conditions in dimensionless form as follows:

$$\begin{aligned} & t' \le 0 : u = 0, \quad \theta = 0, \quad C = 0 \quad for\ all \quad y, \\ & t > 0 : \ u = e^{at}, \ \theta = t, \quad C = t \quad at \quad y = 0, \\ & u \to 0, \theta \to 0, c \to 0 \quad as \quad y \to \infty. \end{aligned} \tag{12}$$

Solution of the problem :

The appeared physical parameters are defined in the nomenclature. The dimensionless governing equations from (9) to (11), subject to the boundary conditions (12) are solved by usual Laplace transform technique and the solutions are expressed in terms of exponential and complementary error functions.

$$\theta(y,t) = \begin{bmatrix} \left(\frac{t}{2} + \frac{y \Pr}{4\sqrt{S}} \right) \exp\left(y\sqrt{S} \right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} + \sqrt{\frac{St}{\Pr}} \right) + \\ \left(\frac{t}{2} - \frac{y \Pr}{4\sqrt{S}} \right) \exp\left(-y\sqrt{S} \right) erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}} - \sqrt{\frac{St}{\Pr}} \right) \end{bmatrix} \tag{13}$$

$$C(y,t) = \left(1 + b\right) \begin{bmatrix} \left(t + \frac{y^2 Sc}{2} \right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} \right) \\ - y\sqrt{\frac{tSc}{\pi}} \exp\left(-\frac{y^2 Sc}{4t} \right) \end{bmatrix} + \left(d - \frac{b}{c} \right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} \right)$$

$$-\frac{1}{2}\left(d - \frac{b}{c} \right) \exp(-ct) \begin{bmatrix} \exp\left(y\sqrt{-cSc} \right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{-ct} \right) \\ + \exp\left(-y\sqrt{-cSc} \right) erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{-ct} \right) \end{bmatrix}$$

$$-\frac{1}{2}\left(d-\frac{b}{c}\right)\begin{bmatrix}\exp(y\sqrt{S})\,erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\dfrac{St}{\Pr}}\right)\\ +\exp(-y\sqrt{S})\,erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\dfrac{St}{\Pr}}\right)\end{bmatrix}$$

$$-b\begin{bmatrix}\left(\dfrac{t}{2}+\dfrac{y\Pr}{4\sqrt{S}}\right)\exp(y\sqrt{S})\,erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\dfrac{St}{\Pr}}\right)\\ +\left(\dfrac{t}{2}-\dfrac{y\Pr}{4\sqrt{S}}\right)\exp(-y\sqrt{S})\,erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\dfrac{St}{\Pr}}\right)\end{bmatrix}$$

$$+\frac{1}{2}\left(d-\frac{b}{c}\right)\exp(-ct)\begin{bmatrix}\exp\left(y\sqrt{S-c\Pr}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\left(\dfrac{S}{\Pr}-c\right)t}\right)\\ +\exp\left(-y\sqrt{S-c\Pr}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\left(\dfrac{S}{\Pr}-c\right)t}\right)\end{bmatrix} \tag{14}$$

$$u(y,t)=\frac{\exp(at)}{2}\begin{bmatrix}\exp\left(y\sqrt{M+a}\right)erfc\left(\dfrac{y}{2\sqrt{t}}+\sqrt{(M+a)t}\right)\\ +\exp\left(-y\sqrt{M+a}\right)erfc\left(\dfrac{y}{2\sqrt{t}}-\sqrt{(M+a)t}\right)\end{bmatrix}$$

$$+A_1\begin{bmatrix}\left(\dfrac{t}{2}+\dfrac{y\Pr}{4\sqrt{S}}\right)\exp\left(y\sqrt{S}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\dfrac{St}{\Pr}}\right)\\ +\left(\dfrac{t}{2}-\dfrac{y\Pr}{4\sqrt{S}}\right)\exp\left(-y\sqrt{S}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\dfrac{St}{\Pr}}\right)\end{bmatrix}$$

$$+A_2\left[\left(t+\frac{y^2Sc}{2}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)-y\sqrt{\frac{tSc}{\pi}}\exp\left(-\frac{y^2Sc}{4t}\right)\right]$$

$$-(A+A_2)\begin{bmatrix}\left(\dfrac{t}{2}+\dfrac{y}{4\sqrt{M}}\right)\exp(y\sqrt{M})\,erfc\left(\dfrac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\\ +\left(\dfrac{t}{2}-\dfrac{y}{4\sqrt{M}}\right)\exp(-y\sqrt{M})\,erfc\left(\dfrac{y}{2\sqrt{t}}-\sqrt{Mt}\right)\end{bmatrix}$$

$$+\frac{A_3}{2}\exp(-ct)\begin{bmatrix}\exp\left(y\sqrt{S-c\Pr}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\left(\dfrac{S}{\Pr}-c\right)t}\right)\\ +\exp\left(-y\sqrt{S-c\Pr}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\left(\dfrac{S}{\Pr}-c\right)t}\right)\end{bmatrix}$$

$$+\frac{A_4}{2}\exp(-ct)\begin{bmatrix}\exp\left(y\sqrt{-cSc}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{-ct}\right)\\ +\exp\left(-y\sqrt{-cSc}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{-ct}\right)\end{bmatrix}$$

$$+\frac{A_5}{2}\exp(-lt)\begin{bmatrix}\exp\left(y\sqrt{M-l}\right)erfc\left(\dfrac{y}{2\sqrt{t}}+\sqrt{(M-l)t}\right)\\ +\exp\left(-y\sqrt{M-l}\right)erfc\left(\dfrac{y}{2\sqrt{t}}-\sqrt{(M-l)t}\right)\end{bmatrix}$$

$$-\frac{A_5}{2}\exp(-lt)\begin{bmatrix}\exp\left(y\sqrt{S-l\Pr}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\left(\dfrac{S}{\Pr}-l\right)t}\right)\\ +\exp\left(-y\sqrt{S-l\Pr}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\left(\dfrac{S}{\Pr}-l\right)t}\right)\end{bmatrix}$$

$$+\frac{A_6}{2}\exp(nt)\begin{bmatrix}\exp\left(y\sqrt{M+n}\right)erfc\left(\dfrac{y}{2\sqrt{t}}+\sqrt{(M+n)t}\right)\\ +\exp\left(-y\sqrt{M+n}\right)erfc\left(\dfrac{y}{2\sqrt{t}}-\sqrt{(M+n)t}\right)\end{bmatrix}$$

$$-\frac{A_6}{2}\exp(nt)\begin{bmatrix}\exp\left(y\sqrt{nSc}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{nt}\right)\\ +\exp\left(-y\sqrt{nSc}\right)erfc\left(\dfrac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{nt}\right)\end{bmatrix}$$

$$+\frac{A_7}{2}\begin{bmatrix}\exp\left(y\sqrt{S}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\dfrac{St}{\Pr}}\right)+\\ \exp\left(-y\sqrt{S}\right)erfc\left(\dfrac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\dfrac{St}{\Pr}}\right)\end{bmatrix}$$

$$+A_8 erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}\right)$$

$$-\frac{1}{2}\left(A_7+A_8\right)\left[\begin{array}{l}\exp(y\sqrt{M})erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\\+\exp(y\sqrt{M})erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{Mt}\right)\end{array}\right] \quad (15)$$

where

$$S=R+H,\ b=S_0Sc,\ c=\frac{S}{\Pr-Sc},$$

$$d=\frac{b\Pr}{S}, l=\frac{S-M}{Sc-1}, n=\frac{M}{Sc-1},$$

$$A_1=\frac{bGm\cos\alpha-Gr\cos\alpha}{S-M}, A_2=\frac{(1+b)Gm\cos\alpha}{M},$$

$$A_3=\frac{bGm\cos\alpha\left(S-c\Pr\right)}{cS\left(S-M+c-c\Pr\right)},$$

$$A_4=\frac{bGm\cos\alpha(S-c\Pr)}{cS(S-M+c-c\Pr)},$$

$$A_5=\frac{(\Pr-1)\left[\begin{array}{l}SGr\cos\alpha(S-M+c-c\Pr)\\+Gm\cos\alpha\, bc(M\Pr-S)\end{array}\right]}{S(S-M)^2(S-M+c-c\Pr)}$$

$$A_6=\frac{Gm\cos\alpha(Sc-1)\left[\begin{array}{l}M(S+bc\Pr)\\+cS(1+b)(Sc-1)\end{array}\right]}{M^2S(M-c+cSc)}$$

$$A_7=\frac{\left[\begin{array}{l}cS(\Pr-1)(Gr\cos\alpha-bGm\cos\alpha)\\+Gm\cos\alpha\, b(S-M)(c\Pr-S)\end{array}\right]}{cS(S-M)^2}$$

$$A_8=\frac{Gm\cos\alpha\left[\begin{array}{l}cS(Sc-1)+M\Pr bc\\-bS(M+c-cSc)\end{array}\right]}{cM^2S}$$

Nusselt number :

From temperature field, now we study Nusselt number (rate of change of heat transfer) which is given in non-dimensional form as

$$Nu=-\left[\frac{\partial\theta}{\partial y}\right]_{y=0} \quad (16)$$

From equations (13) and (16), we get Nusselt number as follows:

$$Nu=\left[t\sqrt{S}erf\sqrt{\frac{St}{\Pr}}+\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{St}{\Pr}\right)+\frac{\Pr}{2\sqrt{S}}erf\sqrt{\frac{St}{\Pr}}\right]$$

Sherwood number :

From concentration field, now we study Sherwood number (rate of change of mass transfer) which is given in non-dimensional form as

$$Sh=-\left[\frac{\partial C}{\partial y}\right]_{y=0} \quad (17)$$

From equations (14) and (17), we get Sherwood number as follows:

$$Sh=2(1+b)\sqrt{\frac{tSc}{\pi}}+\left(d-\frac{b}{c}\right)\sqrt{\frac{Sc}{\pi t}}$$

$$-\left(d-\frac{b}{c}\right)\exp(-ct)\left[\begin{array}{l}\sqrt{\frac{Sc}{\pi t}}\exp(ct)\\+\sqrt{-cSc}erf\sqrt{-ct}\end{array}\right]$$

$$-\left(d-\frac{b}{c}\right)\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{St}{\Pr}\right)+\sqrt{S}erf\sqrt{\frac{St}{\Pr}}\right]$$

$$+\left(d-\frac{b}{c}\right)\exp(-ct)\left[\begin{array}{l}\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{St}{\Pr}+ct\right)\\ +\sqrt{S-c\Pr}\,erf\sqrt{\left(\frac{S}{\Pr}-c\right)t}\end{array}\right]$$

$$-b\left[\begin{array}{l}t\sqrt{S}erf\sqrt{\frac{St}{\Pr}}+\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{St}{\Pr}\right)\\ +\frac{\Pr}{2\sqrt{S}}erf\sqrt{\frac{St}{\Pr}}\end{array}\right]$$

Results and Discussion

In order to get the physical insight into the problem, we have plotted velocity, temperature, concentration, the rate of heat transfer and the rate of mass transfer for different values of the physical parameters like Radiation parameter (R), Magnetic parameter(M), Soret number (So), Schmidt number (Sc), Thermal Grashof number (Gr), Mass Grashof number (Gm), time (t) and Prandtl number (Pr) in figures 1 to 14 for the cases of heating (Gr < 0, Gm < 0) and cooling (Gr > 0, Gm > 0) of the plate at time t = 0.4. The heating and cooling take place by setting up free-convection current due to temperature and concentration gradient.

Figure 1 reveals the effect magnetic field parameter on fluid velocity and we observed that an increase in magnetic parameter M the velocity decreases in cases of cooling and heating of the plate for Pr = 0.71. It is due to fact that the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. Figure (2) displays the influence of thermal-diffusion parameter (soret number So) on the velocity field in both cases of cooling and heating of the plate. It is found that the fluid velocity increases with increasing values of So in case of cooling of the plate and a reverse effect is observed in the case of heating of the plate. Figure 3&4 show the effects of Gr (thermal Grashof number) and Gm (mass Grashof number) and time t on the velocity field u. From these figures it is found that the velocity u increases as thermal Grashof number Gr or mass Grashof number Gm or time t increases in case of cooling of the plate. It is because that increase in the values of thermal Grashof number and mass Grashof number has the tendency to increase the thermal and mass buoyancy effect. This gives rise to an increase in the induced flow transport. And a reverse effect is indentified in case of heating of the plate. The variation in velocity for different values of accelerated parameter is exhibited in figure 5. It is seen that the velocity increases as accelerated parameter a increases in both cases of cooling and heating of the plate. From Tables 1-4 it is observed that with the increase of radiation parameter R or heat source parameter H the velocity increases up to certain y value (distance from the plate) and decreases later for the case of cooling of the plate. But the trend is just reversed in case of heating of the plate. In all above investigations, it is also observed that the fluid has higher velocity when the surface is vertical (α=0) than when inclined (α=π/4) in case of cooling, because of the fact that the buoyancy effect decreases due to gravity component (g cosα), as the plate is inclined. For velocity field, finally from figure 6 it is seen that the fluid (air) velocity is decreased for increasing in case of cooling problem and an opposite phenomenon is observed in case of heating.

The temperature of the flow field is mainly affected by the flow parameters, namely, Radiation parameter (R) and the heat source parameter (H). The effects of these parameters on temperature of the flow field are shown in figures 7. It is observed that as radiation parameter R or heat source parameter H increases the temperature of the flow field decreases at all the points in flow region.

The concentration distributions of the flow field are displayed through figures 8, 9 & 10. It is affected by three flow parameters, namely Soret number (So), Schmidt number (Sc) and radiation parameter(R) respectively. From figure 8 it is observed that the concentration increases with an increase in So (Soret number). Figure 9 & 10 reveal the effect of Sc and R on the concentration distribution of the flow field. The concentration distribution is found to increase faster up to certain y value (distance from the plate) and decreases later as the Schmidt parameter (Sc) or Radiation parameter (R) become heavier.

Nusselt number is presented in Figure 11 against time t. From this figure the Nusselt number is observed to increase with increase in R for both water (Pr=7.0) and air (Pr=0.71). It is also observed that Nusselt number for water is higher than that of air (Pr=0.71). The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities and therefore heat is able to diffuse away from the plate more rapidly than higher values of Pr, hence the rate of heat transfer is reduced. Finally, from figure 12 it is seen that the Sherwood number decreases with increase in Sc (Schmidt number), So (soret number) and R (radiation parameter).

GRAPHS:

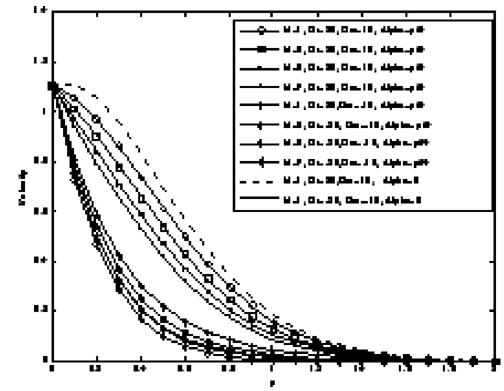

Figure 1. Velocity profiles for different M with so=5, Sc=2.01, Pr=0.71, R=10, H=2, a=0.5 and t=0.2

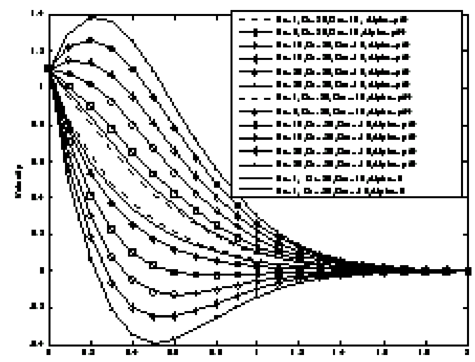

Figure 2. Velocity profiles for different So with M=3, Sc=2.01, Pr=0.71, R=10, H=2, a=0.5 and t=0.2

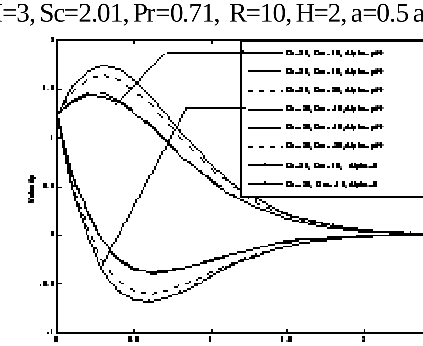

Figure 3. Velocity profiles for different Gr and Gm with so=5, Sc=2.01, M=3, R=10, H=4, a=0.5, Pr=0.71 and t=0.4

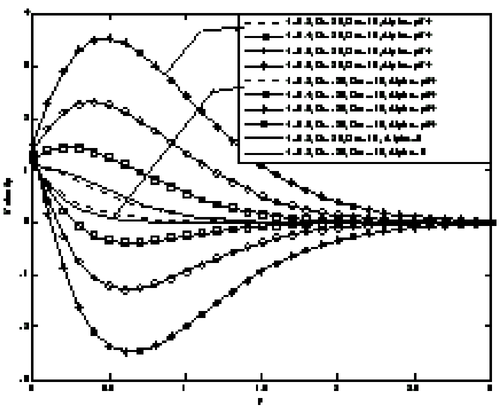

Figure 4. Velocity profiles for different time t with so=5, M=3, Pr=0.71, R=10, H=2, a=0.5, t=0.4

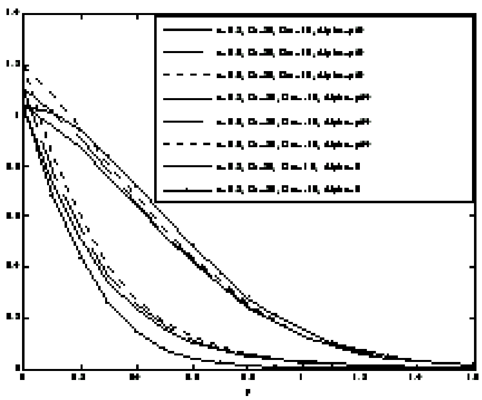

Figure 5. Velocity profiles for different a when So=5, Sc=2.01, M=3, Pr=0.71, R=10, H=2, t=0.2

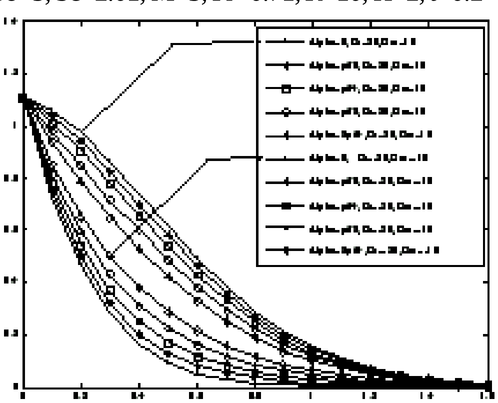

Figure 6. Velocity profiles for different with M=3, Sc=2.01, Pr=0.71, R=10, H=2, a=0.5, t=0.2

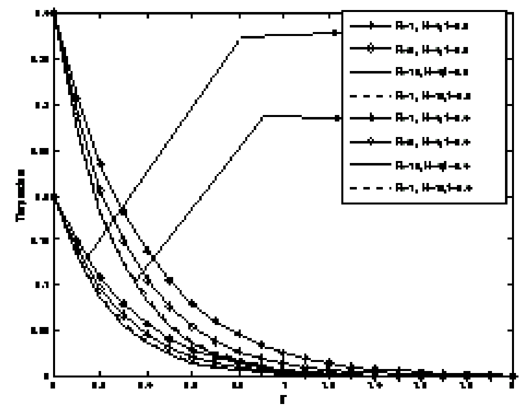

Figure 7. Temperature profiles for different R and H

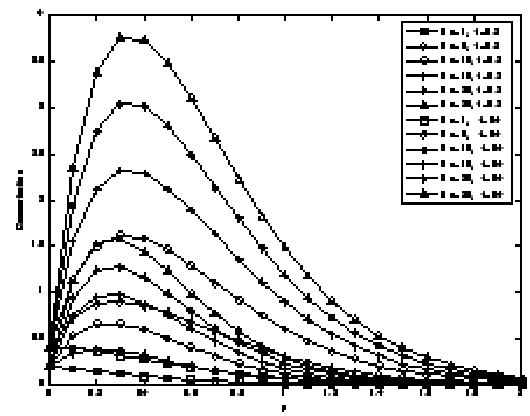

Figure 8. Concentration profiles for different So with R=4, H=1, Sc=2.01 and Pr=0.71

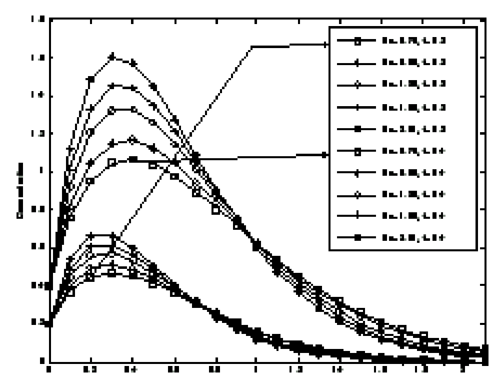

Figure 9. Concentration profiles for different Sc with So=5, Pr=0.71, R=4 and H=1

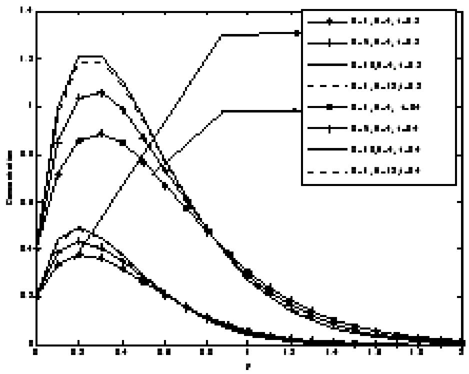

Figure 10. Concentration profiles for different R with So=5, Sc=2.01, H=1 and Pr=0.71

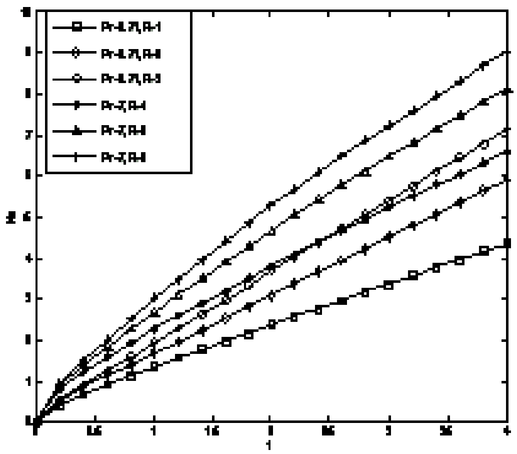

Figure 11. Nusselt Number

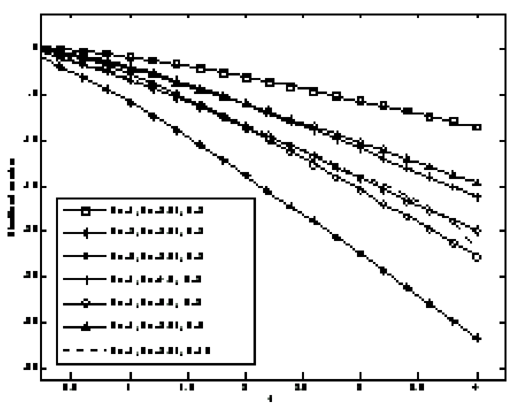

Figure 12. Sherwood number for different Sc, So and R

Table 1. Velocity for different R for Gr=15, Gm=10 (cooling of the plate) with So=5, Sc=2.01, Pr=0.71, M=3, H=4, a=0.5 and t=0.2

	$\alpha = \pi/4$				$\alpha = 0$	
y	R=2	R=4	R=6	R=8	R=2	R=4
0.0	1.1052	1.1052	1.1052	1.1052	1.1052	1.1052
0.2	0.8876	0.8918	0.8957	0.8994	0.9567	0.9626
0.4	0.6469	0.6494	0.6518	0.6540	0.7276	0.7312
0.6	0.4264	0.4257	0.4252	0.4247	0.4915	0.4906
0.8	0.2553	0.2528	0.2505	0.2486	0.2984	0.2949
1.0	0.1394	0.1367	0.1344	0.1323	0.1643	0.1605
1.2	0.0697	0.0676	0.0659	0.0644	0.0826	0.0797
1.4	0.0320	0.0307	0.0297	0.0288	0.0381	0.0363
1.6	0.0135	0.0128	0.0123	0.0118	0.0162	0.0152
1.8	0.0053	0.0049	0.0047	0.0045	0.0063	0.0059
2.0	0.0019	0.0018	0.0016	0.0016	0.0023	0.0021

Table 2. Velocity for different R for Gr=-15, Gm=-10 (Heating of the plate) with So=5, Sc=2.01, Pr=0.71, M=3, H=4, a=0.5 and t=0.2

	$\alpha = \pi/4$				$\alpha = 0$	
y	R=2	R=4	R=6	R=8	R=2	R=4
0.0	1.1052	1.1052	1.1052	1.1052	1.1052	1.1052
0.2	0.5543	0.5501	0.5462	0.5426	0.4853	0.4793
0.4	0.2569	0.2543	0.2520	0.2498	0.1761	0.1725
0.6	0.1121	0.1128	0.1133	0.1138	0.0470	0.0479
0.8	0.0469	0.0494	0.0517	0.0536	0.0038	0.0073
1.0	0.0191	0.0218	0.0241	0.0262	-0.0059	-0.0020
1.2	0.0075	0.0095	0.0113	0.0128	-0.0054	-0.0025
1.4	0.0027	0.0040	0.0051	0.0060	-0.0034	-0.0015
1.6	0.0009	0.0015	0.0021	0.0026	-0.0018	-0.0008
1.8	0.0002	0.0005	0.0008	0.0010	-0.0009	-0.0004
2.0	0.0000	0.0001	0.0003	0.0003	-0.0004	-0.0002

Table 3. Velocity for different H for Gr=15, Gm=10 (cooling of the plate) with So=5, Sc=2.01, Pr=0.71, M=3, R=10, a=0.5 and t=0.2

	$\alpha = \pi/4$				$\alpha = 0$	
y	H=2	H=4	H=6	H=8	H=2	H=4
0.0	1.1052	1.1052	1.1052	1.1052	1.1052	1.1052
0.2	0.9028	0.9060	0.9090	0.9118	0.9781	0.9826
0.4	0.6560	0.6578	0.6595	0.6611	0.7405	0.7431
0.6	0.4242	0.4238	0.4234	0.4231	0.4884	0.4878
0.8	0.2469	0.2454	0.2440	0.2428	0.2866	0.2844
1.0	0.1306	0.1290	0.1276	0.1264	0.1518	0.1496
1.2	0.0631	0.0620	0.0611	0.0602	0.0733	0.0717
1.4	0.0280	0.0274	0.0268	0.0263	0.0324	0.0315
1.6	0.0114	0.0111	0.0108	0.0106	0.0132	0.0127
1.8	0.0043	0.0041	0.0040	0.0039	0.0049	0.0047
2.0	0.0015	0.0014	0.0014	0.0013	0.0017	0.0016

Table 4. Velocity for different H for Gr=-15, Gm=-10 (Heating of the plate) with So=5, Sc=2.01, Pr=0.71, M=3,R=10, a=0.5 and t=0.2

	$\alpha = \pi/4$				$\alpha = 0$	
y	H=2	H=4	H=6	H=8	H=2	H=4
0.0	1.1052	1.1052	1.1052	1.1052	1.1052	1.1052
0.2	0.5391	0.5360	0.5330	0.5301	0.4638	0.4593
0.4	0.2478	0.2459	0.2442	0.2426	0.1632	0.1606
0.6	0.1143	0.1147	0.1151	0.1154	0.0501	0.0507
0.8	0.0553	0.0568	0.0582	0.0594	0.0156	0.0178
1.0	0.0279	0.0295	0.0308	0.0321	0.0067	0.0088
1.2	0.0141	0.0152	0.0161	0.0170	0.0039	0.0054
1.4	0.0067	0.0074	0.0079	0.0084	0.0023	0.0032
1.6	0.0030	0.0033	0.0035	0.0038	0.0012	0.0016
1.8	0.0012	0.0013	0.0014	0.0015	0.0005	0.0007
2.0	0.0004	0.0005	0.0005	0.0006	0.0002	0.0003

References

1. M. S. Alam, M. M. Rahman and M. A. Maleque, Local similarity solutions for unsteady MHD free convection and mass transfer flow past an impulsively started vertical porous plate with Dufour and Soret effects, *Thammasat Int. J. Sci. Tech. 10(3),* 1-8 (2005).
2. M.S.Alam, M.M. Rahman and M.A. Samad, Numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion, Nonlin. *Anal. Model. Control 11(4),* 331-343 (2006).
3. M.M. Alam and M.A. Sattar, Transient MHD heat and mass transfer flow with thermal diffusion in a rotating system, *J. Energy Heat Mass trans.* 21m, 9-21 (1999).
4. U.N. Das, R.K. Deka and V.M. Soundalgekar, Radiation effects on flow past an impulsively started vertical infinite plate, *J.theo. Mech. 1,* 111-115 (1996).
5. M.A. Hossain and H.S. Takhar, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, *Heat Mass Trans. 31,* 243-248 (1996).
6. B. K. Jha and A. K. Singh, Soret effects on free-convection and mass transfer flow in the Stokes problem for a infinite vertical plate, *Astrophys. Space Sci. 173(2),* (1990).
7. N.G. Kafoussias, MHD thermal –diffusion effects on free convective and mass transfer flow over an infinite vertical moving plate. *Astrophys. Space Sci. 192(1),* 11-19 (1992).
8. M. Kumari and G. Nath, Development of two-dimensional boundary layer with an applied magnetic field due to an impulsive motion, I*ndian J. Pure Appl. Math. 30,* 695-708 (1999).
9. R. Muthucumaraswamy and B. Janakiraman, MHD and radiation effects on moving isothermal vertical plate with variable mass diffusion, *Theo. Appl. Mech. 33(1),* 17-29 (2006).
10. A. Raptis and C. Perdikis, Radiation and free convection flow past a moving plate, *Int.J.Appl Mech. Eng. 4,* 817-821 (1999).
11. V.M. Soundalgekar, S.K. Gupta and N.S. Birajdar, Effects of mass transfer and free convection currents on MHD Stokes problem for a vertical plate, *Nuclear Eng. Des. 53,* 339-346 (1979).
12. V.M. Soundalgekar, M.R. Patil and M.D. Jahagirdar, MHD Stokes problem for a vertical plate with variable temperature, *Nuclear Eng. Des. 64,* 39-42 (1981).
13. V.M. Soundalgekar and H.S. Takhar, Radiation effects on free convection flow past a semi-infinite vertical plate, *Model. Measure. Comrol,* 31-40 (1993).
14. V. Rajesh and S.V.K. Varma, Thermal diffusion and radiation effects on MHD flow past an infinite vertical plate with variable temperature and mass diffusion, *JP Journal of Heat and Mass Transfer,* Vol. *3,* Issue1, Feb (2009).
15. A.G.V. Kumar and S. Vijaya Kumar Varma, Thermal diffusion and Radiation effects on unsteady MHD flow past an impulsively started exponentially accelerated vertical plate with variable temperature and variable mass diffusion, *Int. J. of Applied Mathematical Analysis and Applications,* Vol.*6,* No.*1-2,* Jan-Dec., pp.169-192 (2011).