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Change of entropy for galaxies clustering

J.P. SINGH, SNEH LATA* and A.N. PANDEY

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Abstract

We present the entropy change in gravitational clustering of galaxies in view of thermodynamical and statistical scheme in point mass particles system and extended mass particles (non-point mass) *i.e.* in an expanding universe. We obtain analytically the relations for gravitational entropy in terms of temperature T and average density $\bar{n}$ of galaxies (particles) in the given phase cell.

Introduction

Clusters and group of galaxies are known gravitationally bound objects which come in picture in the process of the structure formation as presented by Voit[8]. They are responsible to form the most dense part of the large scale structure of the universe. The clusters are associated with larger groups known as super-clusters. Clusters of galaxies are most massive objects in the structure formation of the universe and their study provides the proper mechanism for the formation of galaxies and their evolution. The average density $\bar{n}$ and temperature T relate some thermal history of cluster formation of a gravitating system. The concept of entropy is not generally well understood though it is measure of how disorganised system is to explain entropy microscopically is very challenging both from the theoretical and experimental point of view. According to thermodynamics entropy s of a system is related with absolute temperature T as $\delta E = T = \delta s$ or,

$$\frac{1}{T} = \frac{\partial S}{\partial E} \tag{1}$$

According to statistical mechanics, one obtains-

$$\frac{1}{T} = k\beta = k\frac{\partial}{\partial E}\left(\log\Omega\right) \tag{2}$$

In view of equation (1) and (2), we get

$$\frac{\partial S}{\partial E} = k\frac{\partial}{\partial E}\left(\log\Omega\right) \tag{3}$$

Integrating one obtains

$$S = k\log\Omega \tag{4}$$

This is the relation between entropy and probability and is known as Boltzmann statistical definition for entropy. Frampton[2] have shown that the entropy of our universe is dominated by black holes, whose entropy is of the order of their area in Planck units. It is very difficult to measure the entropy of the system in real experiments.

* Present Address : D/o Atma Ram Mishra, Nandan Niwas, Hazaribagh, Mau Road, Sidhari, Azamgarh (U.P.) Mob. 9450821510

We have presented the applicability of thermodynamics and statistical mechanics to evaluate the change of entropy $S - S_\circ$ where $S_\circ$ being constant, for gravitating system *i.e.* point mass particles and for extended mass structures in view of the measuring correlation parameter $b = 0$ to 1.

Thermodynamical Description of Galaxy Clustering : The expression for the entropy of the ensemble or assembly reads

$$S = k \log \Omega_{\max} = Nk \log z + \frac{E}{T} \tag{5}$$

Where Z be partition function as

$$Z = \frac{V}{h^3}(2\pi mkT)^{2/3} \tag{6}$$

for a gas molecule; and

$$\log \Omega_{\max} = N \log Z + \beta E$$

where $\beta = \dfrac{1}{kT}$ (7)

In view of the above, we get

$$E = \frac{3}{2} NkT \tag{8}$$

$$S = Nk \log Z + \frac{3}{2} Nk \tag{9}$$

The average energy of the system

$$\overline{\in} = E / N, \tag{10}$$

or $$E = N \overline{\in} = NkT^2 \left[\frac{\partial \log Z}{\partial T}\right]_V \tag{11}$$

The grand partition function Z reads

$$\tau \log Z = \Omega \tag{12}$$

Where $\Omega = -\bar{n}\tau \qquad \tau = kT$ (13)

$$\bar{n} = -\left(\frac{\partial \Omega}{\partial \mu}\right)_{V,\tau} \tag{14}$$

Where μ be the chemical potential. The statistical entropy σ reads

$$\sigma = \bar{n}\left[\log\left\{\left(\frac{2nm\tau}{h^2}\right)^{3/2} \cdot \frac{V}{n}\right\} + \frac{5}{2}\right] \tag{15}$$

Hence, thermodynamical entropy

$$S = k\sigma$$

$$= \bar{n}k \log\left[\left(\frac{2nmkT}{h^2}\right)^{3/2} - \frac{V}{n}\right] + \frac{5}{2}\bar{n}k \tag{16}$$

The classical partition function Z for a canonical ensemble gives

$$Z = \frac{VN}{\text{L}N}\left(\frac{2\pi mkt}{h^2}\right)^{3N/2} \tag{17}$$

Let use define Helmholtz energy F as

$$F = -kT \log Z$$

$$= -kT \log\left(\frac{f}{N}\right) - NkT \tag{18}$$

where $f = V\left(\dfrac{2\pi mkT}{h}\right)^{3/2}$ (19)

We now define entropy S as-

$$S = -\left(\frac{\partial F}{\partial T}\right)_V$$

$$= Nk \log_e\left[\frac{V}{Nh^3}(2\pi mkT)^{3/2} . e^{5/2}\right] \tag{20}$$

In view of the above we get

$$Z = \frac{V^N}{\text{L}N}\left(\frac{2\pi mkT}{h^2}\right)^{3N/2} \tag{21}$$

Let us consider the thermodynamic description for the universe. The universe is taken to be an infinite gas molecules where each gas molecule is treated to be a galaxy. Itoh[5] presented the physical validity of the application of thermodynamics in the galaxy clustering on the basis of N body computer simulation results.

The gravitational force is a binary interaction. Hill[3] present internal energy U and pressure p as

$$U = \frac{3NT}{2}(1-2b), \quad (22)$$

$$p = \frac{NT}{V}(1-b), \quad (23)$$

Where b as the correlation parameter which is dimensionless as shown by Saslaw and Hamilton[7].

$$b = -\frac{W}{2K}$$

$$= 2\tau G m^2 \frac{n}{3T}\int_0^\infty \xi(\bar{n},T,r)rdr \quad (24)$$

where W as the potential energy, and K the kinetic energy of the particles in a system. Let us define the average number density of system of particles each of mass m, T temperature, V the volume and G as gravitational constant.

$$\bar{n} = N/V, \quad (25)$$

Where $\xi(\bar{n},T,r)$ be the two particle correlation function with r as the inter-particle distance. Iqbal[4] studies $\xi(\bar{n},T,r)$. It is obvious for an ideal gas $b=0$ whereas for non-ideal gas b varies between 0 and 1. Iqbal[4], Saslaw and Hamilton[7], obtained b as

$$b = \frac{\beta\bar{n}T^{-3}}{1+\beta\bar{n}T^{-3}} \quad (26)$$

or

$$\bar{n} = \frac{b}{(1-b)\beta T^{-3}} \quad (27)$$

Entropy of the Universe : There are two important ways to describe the entropy of a system (universe) as thermodynamics and statistical mechanics. In view of Ahmad[1], the grand canonical partition function read

$$Z_N(T,V) = \frac{V^N}{\text{L}N}\left(\frac{2nmkT}{h^2}\right)^{3N/2}\left(1+\beta\bar{n}T^{-3}\right)^{N-1} \quad (28)$$

Where N is taken as number of galaxies, h as the volume of a phase cell, with point mass approximation. New the Helmholtz free energy F

$$F = -N\ell n Z_N \quad (29)$$

Hence, thermodynamic entropy may be evaluated as

$$S = -\left(\frac{\partial F}{\partial T}\right)_{N,V} \quad (30)$$

From equations (28) and (20), the equation (30) assumes the form $\delta S =$

$$S - S_\circ = \ell n\left(\bar{n}^{-1}T^{3/2}\right) - \ell n(1-b) - 3b \quad (31)$$

where $S_\circ$ as the arbitrary constant. In view of equation (27), one obtain

$$\delta S = S - S_\circ = -\left(\ell nbT^{3/2} + 3b\right) \quad (32)$$

In view of equation (26), we get

$$T^{3/2} = \left[\frac{\beta\bar{n}(1-b)}{b}\right]^{1/2} \quad (33)$$

using equation (33) in equation (32), one obtains

$$\delta S = S - S_\circ = -\left[\frac{1}{2}\ell n\bar{n} + \frac{1}{2}\ell n[b(1-b)] + 36\right] \quad (34)$$

for an ideal gas $b=0,$ so we get

$$\delta S = S - S_\circ = -\frac{1}{2}\ell n\bar{n} = -\frac{1}{2}\ell n\left(\frac{N}{V}\right) \quad (35)$$

The equation (34) gives an expression for entropy of a system (universe) consisting of point mass particles. Indeed, galaxies have extended structure. Hence, the point mass view is only approximation. For the extended structure, one may put $b_\in$ as

$$b_\in = \frac{\beta\bar{n}T^{-3}\alpha\left(\in/R\right)}{1+\beta\bar{n}T^{-3}\alpha\left(\in/R\right)} \quad (36)$$

where $\in$ as the softening parameter lying between 0.01 and 0.05. There is a relation between b and $b_{\in}$

$$b_{\in}\left[1+b(\alpha-1)\right]=b\alpha\ , \qquad (37)$$

or
$$b_{\in}=\frac{ab}{1+b(1-\alpha)}, \qquad (38)$$

where $b_{\in}$ as the correlation parameter for extended mass particles clustering gravitationally. In view of equation (36), we get

$$T^{3/2}=\left[\frac{\beta\bar{n}\,\alpha\left(\in/R\right)\left(1-b_{\in}\right)}{b_{\in}}\right]^{1/2} \qquad (39)$$

From equation (38) and $a=1$, one obtain

$$b_{\in}=b \qquad (40)$$

Hence, equation (39) assumes the form

$$T^{3/2}=\left[\frac{\beta\bar{n}(1-b)}{b}\right]^{\frac{1}{2}}, \qquad (41)$$

where
$$\alpha\left(\frac{\in}{R}\right)=\sqrt{1+\left(\frac{\in}{R}\right)^{2}}+\left(\frac{\in}{R}\right)^{2}\ell n\frac{\in/R}{1+\sqrt{1+\left(\in/R\right)^{2}}} \qquad (42)$$

where α as constant, R be the radius of the cell in phase space where galaxies are N. Using the same procedure, one may obtain for the extended structures of galaxies.

$$\delta S=S-S_{\circ}$$
$$=-\left[\frac{\ell nbT^{3/2}}{1+b(\alpha-1)}+\frac{3b\alpha}{1+b(\alpha-1)}\right] \qquad (43)$$

and $\delta S=S-S_{\circ}$

$$=-\left[\frac{1}{2}\ell n\bar{n}+\ell n\frac{\left[b(1-b)\right]^{1/2}}{1+b(\alpha-1)}+\frac{3b\alpha}{1+b(\alpha-1)}\right] \qquad (44)$$

In view of equation (42), if we assume the value of $\in$ between0.01 to 0.05 and R, between 0.04 and 0.06, so,

$$\alpha\left(\in/R\right)\approx 1 \qquad (45)$$

Hence, for $\in=0$ and $\alpha=1$ the entropy for point mass and extended mass galaxies are the same.

Conclusion Remarks

We have presented the entropy charge in gravitational clustering of galaxies by using thermodynamical and statistical approach in point-mass particles system and extended mass particles (non-point mass) *i.e*. in an expanding universe. We obtained analytically and relations for gravitational entropy in terms of temperature T and average density $\bar{n}$ of the galaxies (particles) in the given phase cell. The present study provides that the entropy for a system consisting of extended mass particles has similar behaviour then that of point mass particles clustering gravitational in an expanding universe. It is observed that the entropy changes in gravitational galaxy clustering in an expanding universe. It is shown that during the starting stage of clustering of galaxies, the entropy decreases and appears increasing when the system attains equilibrium. It is suggested that the universe is different, which we observe.

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