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MOD n-Cognitive Maps Model

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Abstract

In this paper the new notion of MOD natural neutrosophic numbers Cognitive Maps (MOD-n-CMs) model which is based on MOD natural neutrosophic numbers and Neutrosophic Cognitive Maps (NCMs) is introduced. This model will find it various applications in the fields of engineering and science where indeterminacy plays a role.

Key words : Neutrosophic Cognitive Maps; Neutrosophy; Neural Networks; MOD natural neutrosophic numbers; MOD n-Cognitive Maps; MOD natural neutrosophic directed graph; MOD hidden pattern; MOD natural neutrosophic matrix

I. Introduction

Fuzzy Cognitive Maps (FCMs) model and Neutrosophic Cognitive Maps (NCMs) model are popular among researchers in almost every field of science, medicine and engineering[5,6,8]. However when we consider the models as simple the edge weights are from {0, 1} or {0, 1, I} respectively. So in this situation only the indeterminate I is used and I satisfies $I^2 = I$.

But it was a problem suggested by Smarandache who wanted the neutrosophy or indeterminate to be diverse as zero indeterminate, idempotent indeterminate, nilpotent indeterminate and zero divisor indeterminate. This problem was answered in[7], where the mod integer indeterminates also known as natural neutrosophic elements can cater to all such situations.

Now in this paper mod natural neutrosophicnumbers cognitive maps models using Z_n^I $(2 \leq n \leq \infty)$ is constructed. For more about Z_n^I refer[7]. In order to build such new models the notion of MOD natural neutrosophic directed graphs and MOD natural neutrosophic matrices is needed.

This paper is organized into five sections. Section one is introductory in nature. Section two recalls the notion of MOD natural neutrosophicnumbers (MOD-n). Section three defines the notion of MOD natural neutrosophic directed graphs and illustrates them with examples. Section four defines the new MOD natural neutrosophicnumbers Cognitive Maps (MOD-n-CMs) model and illustrates it. Final section gives the conclusion based on our study.

II. *Mod Natural Neutrosophic Numbers and Their Properties :*

Clearly $\{Z_n, +, \times\}$ is a commutative finite ring of order n. Take $Z_2 = \{0, 1\}$. $\frac{1}{1} = 1$ but $\frac{1}{0}$ is not defined and $\frac{0}{1} = 0$ and $\frac{0}{0}$ is not defined. So if we define the operation of division clearly $Z_2^I = \{1, 0, \frac{1}{0}, \frac{0}{0}\} = \{1, 0, I_0^2\}$ where $I_0^2 = \frac{1}{0} = \frac{0}{0}$ that is any element divided by 0 in Z_2 is an indeterminate and is denoted by I_0^2. They will be known as natural neutrosophic numbers and I_0^n in particular is the natural neutrosophic zero.

Thus $\frac{I_0^2}{I_0^2} = I_0^2$ (by definition).

This is just like $\frac{0}{n}$ ($n \neq 0$ for all $n \in Z \setminus \{0\}$ is defined as 0).

Thus $\{Z_2^I, /\}$ has the following table:

Table 1$\{Z_2^I, /\}$

/	0	1	I_0^2
0	I_0^2	0	I_0^2
1	I_0^2	1	I_0^2
I_0^2	I_0^2	I_0^2	I_0^2

This is the way operation of division is performed on Z_2^I.

$$\frac{0}{I_0^2} = I_0^2,\ \frac{I_0^2}{0} = I_0^2,\ \frac{I_0^2}{I_0^2} = I_0^2,\ \frac{1}{I_0^2} = I_0^2 \text{ and } \frac{I_0^2}{1} = I_0^2.$$

Clearly / is a non commutative operation on Z_2^I.

Now consider $Z_3 = \{0, 1, 2\}$, $Z_3^I = \{0, 1, 2, I_0^3\}$ is again a closed under /. The table for Z_3^I is as follows:

/	0	1	2	I_0^3
0	I_0^3	0	0	I_0^3
1	I_0^3	1	2	I_0^3
2	I_0^3	2	1	I_0^3
I_0^3	I_0^3	I_0^3	I_0^3	I_0^3

Thus I_0^3 and I_0^2 are the natural neutrosophic elements or naturally neutrosophic elements of Z_3^I and Z_2^I respectively.

Now consider $Z_4 = \{0, 1, 2, 3\}$. $Z_4^I = \{0, 1, 2, 3, I_0^4, I_2^4\}$ for $\frac{1}{2}$ is not defined so is $\frac{3}{2}$. $\frac{1}{0}$ is not defined and $\frac{2}{0}, \frac{3}{0}$ are all not defined and they are denoted by I_0^4. $\frac{1}{2}, \frac{3}{2}, \frac{2}{2}, \frac{0}{2}$ are denoted by I_2^4.

$$I_2^4 \times I_2^4 = I_0^4;\ I_2^4 \times I_0^4 = I_0^4 = I_0^4 \times I_2^4$$

$$I_0^4 \times I_0^4 = I_0^4.$$

Thus Z_4^I has two indeterminates I_0^4 and I_2^4.

They are natural neutrosophic elements of Z_4.

Thus if in Z_n, n is not a prime we may have more than one natural neutrosophic element.

Clearly Z_4 has two natural neutrosophicnumbers.

Next for $Z_5 = \{0, 1, 2, 3, 4\}$the natural neutrosophic elements is $Z_5^I = \{0, 1, 2, 3, 4, I_0^5\}$. Thus $o(Z_5^I) = o(Z_4^I)$ but they are not isomorphic.

Consider $Z_6 = \{0, 1, 2, 3, 4, 5\}$, $Z_6^I = \{0, 1, 2, 3, 4, 5, I_0^6, I_2^6, I_4^6, I_3^6\}$. Clearly $3 \in Z_6$ is such that $3^2 = 3$ but also$3 \times 2 = 0$ so 3 is a zero divisor hence $\frac{i}{3}$; $i \in Z_6$ are all indeterminates.

Hence $|Z_6^I| = 10$ and Z_6 contributes to 4 natural neutrosophic numbers. $I_0^6 \times I_2^6 = I_0^6 = I_4^6 \times I_0^6 = I_3^6 \times I_0^6$, $I_2^6 \times I_3^6 = I_0^6$, $I_3^6 \times I_3^6 = I_3^6$, $I_4^6 \times I_3^6 = I_0^6$, $I_4^6 \times I_2^6 = I_2^6$, $I_2^6 \times I_2^6 = I_4^6$, $I_4^6 \times I_4^6 = I_4^6$,

This is the way natural neutrosophic product is defined.

Now product can be defined; addition can be made only in a very special way. Once we write Z_n^I it implies Z_n^I contains all natural neutrosophic numbers from Z_n.

Now in case of $Z_2^I = \{0, 1, I_0^2\}$ if we have to define + operation then the set G =$\{Z_2^I, +\} = \{0, 1, I_0^2, 1 + I_0^2\}$ is only a semigroup under + modulo 2 as $I_0^2 + I_0^2 = I_0^2$ (is defined) and G will be known as natural neutrosophic semigroup.

$\{Z_2^I, \times\} = \{0, 1, I_0^2, \times\}$ is given by the following table.

We define $0 \times I_0^2 = I_0^2$ only and not zero.

TABLE III. $\{Z_2^I, \times\}$

$\times$	0	1	I_0^2
0	0	0	I_0^2
1	0	1	I_0^2
I_0^2	I_0^2	I_0^2	I_0^2

Thus $\{Z_2^I, \times\}$ is a semigroup under $\times$, known as natural neutrosophic product semigroup. $\{Z_2, +, \times\}$ is a field.

Consider $\{Z_3^I, +\} = \{0, 1, 2, I_0^3, 1 + I_0^3, 2 + I_0^3\} = S$. Clearly S is a semigroup under +, as $I_0^3 + I_0^3 = I_0^3$ so nothing will make them equal to zero for $I_0^3 + I_0^3 + I_0^3 = I_0^3 \neq 0$.

The table for Z_2^I under $\times$ is as follows:

Table IV. $\{Z_3^I, \times\}$

$\times$	0	1	I_0^2	$1+I_0^2$
0	0	0	I_0^2	I_0^2
1	0	1	I_0^2	$1+I_0^2$
I_0^2	I_0^2	I_0^2	I_0^2	I_0^2
$1+I_0^2$	I_0^2	$1+I_0^2$	I_0^2	$1+I_0^2$

For more refer[7].

III. MOD Natural Neutrosophic Numbers Directed Graphs and MOD Natural Neutrosophic Matrix :

In this section for the first time the notion of MOD natural neutrosophic directed graphs with edge weights taken from Z_n^I $(2 \leq n < \infty)$ are defined and described.

Definition 3.1: Let G be a directed graph with vertices *$v_1, \ldots, v_t$. If the edge weights are taken from the set Z_n^I then G is defined as the MOD natural neutrosophic numbers directed graph with edge weights from Z_n^I $(2 \leq n < \infty)$.*

We will describe this situation by an example.

Example 3.1: Let G be the MOD natural neutrosophic directed graph with vertices $v_1, v_2, \ldots, v_7$ and the edge weights from Z_6^I. The graph G is given in Fig. 1.

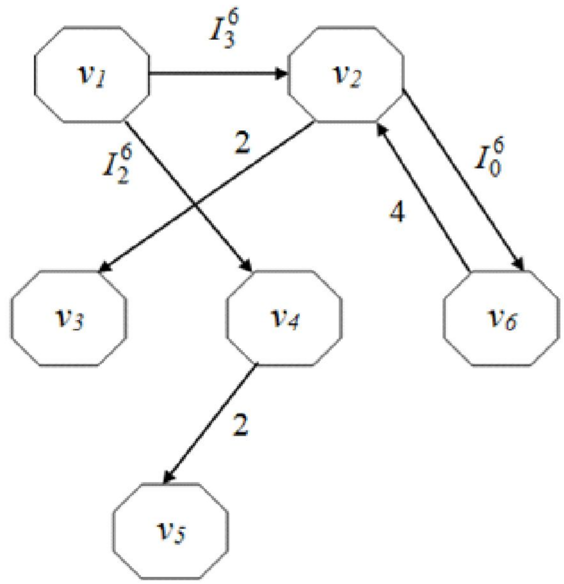

Fig. 1. MOD natural neutrosophic directed graph G

The adjacency matrix M associated with G in (1).

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & I_3^6 & 0 & I_2^6 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & I_0^6 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}. \quad (1)$$

Next the notion of MOD natural neutrosophic matrix is given.

Definition 3.2: Let $M = (m_{ij})$ be a matrix with entries from Z_n^I; that is $m_{ij} \in Z_n^I$; then we define M to be the MOD natural neutrosophic matrix:

M given in (1) is an example of MOD natural neutrosophic matrix with entries from Z_6^I.

Since the aim of this paper is to define MOD natural neutrosophicnumbers Cognitive Maps (MOD-n-CMs) model we do not indulge in describing them.

Here we proceed on to describe the new MOD natural neutrosophicnumbers Cognitive Maps model.

IV. MOD Natural Neutrosophic Numbers Cognitive Maps (MOD-n-CMs) model :

In this section we for the first time introduce this new model and describe the functioning of it. We also illustrate this situation by an example.

Definition 4.1: A MOD natural neutrosophic number cognitive maps is a directed graph with concepts like policies or events etc. as nodes and causalities as edges. It represents MOD causal relation between concepts.

The nodes are taken as fuzzy neutrosophic ones. Here by fuzzy we do not mean it takes values between [0, 1] what we mean is they are terms involving lots of flucating meanings or fuzzy terms and can not be defined; like the complexion of a person or the sweetness of voice of a person or performance aspects of a student studying in a college. For the nodes can be zero indeterminate, nilpotent indeterminate, idempotent indeterminate and zero divisor indeterminate. That is why we call them fuzzy neutrosophic nodes.

Definition 4.2: MOD natural neutrosophic number CMs with edge weights or causalities from the set Z_n^I ($3 \leq n < \infty$) are called simple MOD-n-CMs.

Definition 4.3: Consider the nodes/ concepts $C_1, C_2, \ldots, C_m$ of the MOD-n-CMs. Suppose MOD naturalneutrosophic directed graph is drawn with edge weights $e_{ij} \in Z_n^I$. The MOD matrix E_m be defined by $E_m = (e_{ij})$ where e_{ij} is the weight of the directed edge C_iC_j. E_m is defined as the adjacency MOD natural neutrosophic numbers matrix of the MOD-n-CMs, also known as the connection MOD matrix of the MOD-n-CMs.

Definition 4.4: Let $C_1, C_2, \ldots, C_m$ be the nodes of a MOD-n-CMs. $A = (a_1, a_2, \ldots, a_m)$ where $a_i \in Z_n^I$; $1 \leq i \leq m$. A is called the instantaneous MOD state vector which can denote any of the values from Z_n^I. It may be natural neutrosophic or real or 0 from Z_n^I.

We can also work with the initial MOD vectors $X = \{(x_1, \ldots, x_m) \mid x_i = 0 \text{ or } 1; 1 \leq i \leq m\}$, however the mod resultant can be any value from Z_n^I.

Now the concepts about MOD-n-CMs is defined.

Definition 4.5: Let $C_1, C_2, \ldots, C_n$ be the nodes of the mod-n-CM. Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \overrightarrow{C_3C_4}, \ldots, \overrightarrow{C_iC_j}$ be the MOD edges of the MOD-n-CM.

Then the MOD edges form a MOD directed cycle. A MOD-n-CM is said to be MOD cyclic if it possesses a MOD directed cyclic. An MOD-n-CM is said to be MOD acyclic if it does not possess any MOD directed cycle.

Definition 4.6 : A MOD-n-CM with MOD cycles is said to have a MODfeedback. When there is a MOD feedback in the MOD-n-CM i.e. when the causal relations flow through a cycle in a revolutionary manner the MOD-n-CM is called a MODdynamical system.

Definition 4.7: Let $\overrightarrow{C_1C_2}, \overrightarrow{C_2C_3}, \cdots, \overrightarrow{C_{n-1}C_n}$ be MOD cycle, when C_i is switched on and if the causality flows through the edges of a MOD cycle and if it again causes C_i,, that the MOD dynamical system goes round and round. This is true for any node C_i , for i = 1, 2,…, n. The equilibrium state for this MOD dynamical system is called the MOD hidden pattern.

Definition 4.8: If the MOD equilibrium state of a MOD dynamical system is a unique state vector, then it is called a MOD fixed point.

Consider the MOD-n-CM with $C_1, C_2, \ldots, C_n$ as nodes. For example, start the dynamical system by switching on C_1. Assume that the MOD-n-CM settles down with C_1 and C_n on, i.e. the state vector remain as (1, 0,…, 1) this neutrosophic state vector (1,0,…, 0, 1) is called the MOD fixed point.

Definition 4.9: If the MOD-n-CM settles with a neutrosophic state vector repeating in the form $A_1 \rightarrow A_2 \rightarrow \ldots \rightarrow A_i \rightarrow A_1$, then this equilibrium is called a MOD limit cycle of the MOD-n-CMs.

The methodsused to determine the hiddenpattern is described.

Let $C_1, C_2, \ldots, C_n$ be the nodes of an *MOD n*CM, with feedback. Let E be the associated MODadjacency matrix. Let us find the MODhidden pattern when C_1 is switched on when an input is given as the vector $A_1 = (1, 0, 0, \ldots, 0)$, the data should pass through the MODneutrosophic matrix N(E), this is done by multiplying A_1 by the matrix N(E).

Let $A_1N(E) = (a_1, a_2, \ldots, a_n)$; here the threshold operation of the resultant MOD state vector is not done, since the MOD operation takes care if it at each stage.

We update the resulting concept, the concept C_1 is included in the updated vector by making the first coordinate as 1 in the resulting vector. Suppose $A_1N(E) \rightarrow A_2$ then consider A_2 N(E) and repeat the same procedure.

This procedure is repeated till we get a MOD limit cycle or a MOD fixed point.

The MOD-n-CM model is described by an illustrative example to show the functioning of the new model.

For example, suppose an expert wishes to find the performance aspects of students studying in colleges. The expert has the following nodes described briefly.

C_1: Good at studies :

The student is good at studies is assumed by his/her marks at the entrance test and his/her marks in school. It may not at the end of the assessment period or in the middle of the course which reflect his being good in studies that is why the expert wishes to work with MOD-n-CMs model. It can be a value like I_0^n or I_t^n also reflecting the indeterminate factor about his studies by his teachers. Hence at the outset the expert is justified in using in using this new model.

C_2: The student has good communication skills :

It might have been assumed that the student has good communication skills based on his performance at school and the entrance exam. But in the course of time, this could have changed drastically as in the case of C_1.

C_3: Good behaviour in the classroom and on campus :

This is unpredictable because nothing can be said about this. So it can also be a natural neutrosophic number as the character certificate given in schools are basically a formality. Secondly, surroundings and circumstance can change this node, but however it is assumed for the problem that the student has good behavior and study its effect on other nodes.

C_4: Punctual to class and attentive in class :

This node cannot be fixed in a day or two or by any other data.

C_5: Not interested in studies :

This also cannot be determined for a specific reason for it can be due to language problem or communication skill or teachers attitude or his social and economic conditions.

So at the outset as the nodes can be real or 0 or an indeterminate of various types we are justified in using the MOD-n-CMs model with edge weights form Z_6^I.

Here $Z_6^I = \langle\{0,1, 2, 3, 4, 5, I_0^6, I_0^6, I_0^6, I_0^6\}\rangle$ denotes the set is generated under the operation sum + and × product. Let G be the mod directed graph given by the expert with edge weights from Z_6^I as given in Fig 2.

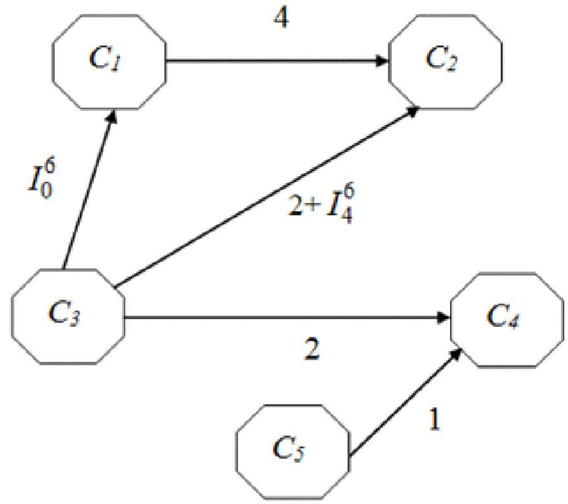

Fig. 2. Mod Directed Graph

The mod connection matrix M associated with the graph G is given in (2).

$$M = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{array}\begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & C_4 & C_5 \end{array} \\ \begin{bmatrix} 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ I_0^6 & 2+I_4^6 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad (2)$$

Let X = $\{(a_1\ a_2\ a_3\ a_4\ a_5) \mid a_i \in \{0, 1\}; 1 \le i \le 5\}$ be the initial state of vectors associated with M.

Let x = (1 0 0 0 0) $\in$ X;

$xM = (0\ 4\ 0\ 0\ 0) \to (1\ 4\ 0\ 0\ 0) = y_1$

$y_1M \to (1\ 4\ 0\ 0\ 0) = y_2\ (= y_1)$.

($\to$ denotes the MOD resultant state vector has been updated).

Thus the MOD resultant is a MOD fixed point.

Let x = (0 1 0 0 0) $\in$ X; then

$xM \to (0\ 1\ 0\ 0\ 0)$.

Thus the MOD resultant is a MOD fixed point.

Let x = (0 0 1 0 0) $\in$ X; to find the effect of x on M.

$xM \to (I_0^6\ \ I_0^6 + 2\ 1\ 2\ 0) = y_1$

$y_1M \to (I_0^6\ \ 2 + I_4^6 + I_0^6\ \ 1\ \ 2\ \ 0) = y_2$

$y_1M \to (I_0^6\ \ 2 + I_4^6 + I_0^6\ \ 1\ \ 2\ \ 0) = y_2\ (= y_1)$.

Thus the MOD resultant is a MOD fixed point. This is only an illustrative model.

From this it is evident behavior in class room makes only the node C_1 as zero indeterminate and also it makes the node C_2 as sum of two indeterminate and a real. All these are realistic for the good behavior of a student need not be imply he is good at studies or has good communicative skills but certainly he is punctual for C_4 takes the real value 2, for he has good behavior. This sort of getting different values that too indeterminate zero values or sum of indeterminates to nodes is an impossibility in FCMs and NCMs.

FCMs can maximum give on or off state of the nodes and NCMs can give on or off or indeterminate state to nodes in any result.

Thus this type of model would certainly be helpful to scientists, engineers and medical researchers who are in need to make use of indeterminacy.

I. Conclusions

In this paper for the first time authors introduce a new model called the MOD natural neutrosophic numbers Cognitive Maps (MOD-n-CMs) model where the dynamical system uses entries from Z_n^I $(3 \le n < \infty)$. The advantages of using this model is

1. The nodes of this model can take any value in which means at that instant for a particular initial state vector the node can be real value in Z_n or zero neutrosophic or nilpotent neutrosophic or idempotent neutrosophic or neutrosophic zero-divisor.
2. Certainly using this model is advantageous because it can give at an instant a value in Z_n^I different from 1 or 0 or I. Secondly the edge weights will also take its values in Z_n^I which is yet another merit of this model.
3. Further we are sure to arrive at a MOD fixed point or a MOD limit cycle after a finite number of iterations.
4. Yet another advantage of using this model is we need not use the notion of thresholding for it may create bais or variations in the experts opinion. The modulo sum takes care of it.

We are sure this new innovative model will find lots of application in the fields of engineering, medical and social sciences.

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