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Variable Heat and Mass Transfer on MHD flow of Visco-elastic fluid past an impulsively started vertical porous plate

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Abstract

In this paper, we have discussed the MHD flow of visco-elastic fluid through a loosely packed porous medium in an impulsively started vertical plate with variable heat and mass transfer. The temperature of plate is made to rise linearly with time. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The equations for the governing flow are solved by making use Laplace-transform technique. The velocity, temperature and concentration are obtained analytically and computationally discussed with reference to governing parameters and are illustrated graphically, and physical aspects of the problem are discussed. Also skin friction, Nusselt number and Sherwood number are obtained analytically and are tabulated.

Keywords : Radiation effects, MHD flows, Heat and mass transfer, vertical plates, porous medium, Skin friction, Nusselt number, Sherwood number.

Nomenclature

β Volumetric coefficient of thermal expansion
β^* Volumetric coefficient of expansion with concentration
σ Stefan–Boltzmann constant
ρ Density
θ Dimensionless temperature
ν Kinematic viscosity
μ Coefficient of viscosity
τ Dimensionless skin friction
b Similarity parameter
a^* Absorption coefficient
A Constant
B_0 External magnetic field
C Species concentration in the fluid
$\overline{C}$ Dimensionless concentration
Cp Specific heat at constant pressure
C_w Concentration of the fluid
C_1 Concentration in the fluid far away from the plate
D_1 Chemical molecular diffusivity
erf Error function
$erfc$ Complementary error function
g Acceleration due to gravity
Gm Mass Grashof number
Gr Thermal Grashof number
D Darcy parameter
α_1 is the normal stress modulus,
k Thermal conductivity of the fluid
M Magnetic field parameter

Nu Dimensional Nusselt number
Pr Prandtl number
q_r Radiative heat flux in the y direction
R Radiation parameter
Sc Schmidt number
Sh Dimensional Sherwood number
T Temperature of the fluid near the plate
t Time
$\bar{t}$ Dimensional time
T_w Temperature of the fluid
T_∞ Temperature of the fluid far away from the plate
u Velocity of the fluid in the x - direction
u_0 Velocity of the fluid
$\bar{u}$ Dimensionless velocity
y Coordinate axis normal to the plate
$\bar{y}$ Dimensionless coordinate axis normal to the plate

Subscripts

w Conditions on the wall
∞ Free stream conditions

1. Introduction

In recent years, a great deal of interest has been created on heat and mass transfer of the boundary layer flow over a stretching sheet, in view of its numerous applications in various fields such as polymer processing industry in manufacturing processes. Crane[1] computed an exact similarity solution for the boundary layer flow of a Newtonian fluid toward an elastic sheet which is stretched with the velocity proportional to the distance from the origin. Sakiadis[2, 3] first studied the boundary layer problem assuming velocity of a bounding surface as constant. The convection problem in porous medium has also important applications in geothermal reservoirs and geothermal energy extractions. A comprehensive review of the studies of convective heat transfer mechanism through porous media has been made by Nield and Bejan[4]. Hiremath and Patil[5] studied the effect on free convection currents on the oscillatory flow through a porous medium, which is bounded by vertical plane surface of constant temperature. Fluctuating heat and mass transfer on unsteady free convective MHD flow through porous media in a rotating system has been discussed by Dash *et al*.[6]. Subhashini *et al*.[7] studied the effect of mass transfer on the flow past a vertical porous plate. Unsteady free convective flow with mass transfer phenomenon past an infinite vertical porous plate with constant suction was studied by Soundalgekar and Wavre[8]. Soundalgekar[9] studied the effects of mass transfer and free convection currents on the flow past an impulsively started vertical plate. In these studies the magnetohydrodynamic effect has been ignored. Lai and Kulacki[10] studied the coupled heat and mass transfer with natural convection from vertical surface in a porous medium. Benazir *et al*.[11] have studied unsteady MHD Casson fluid flow over a vertical cone and flat plate with non-uniform heat source/sink. Kumar *et al*.[12] studied chemically reacting MHDfree convective flow over a vertical cone with a variable electric conductivity. Further, Prakash *et al*.[13] contributed through their publication entitled radiation and Dufour effects on unsteady MHD mixed convective flow in an accelerated vertical wavy plate with varying temperature and mass diffusion. They have considered an unsteady mixed convective flow past a vertically wavy plate with varying temperature and mass diffusion. Sivaraj and Kumar[14] have analyzed unsteady MHD dusty viscoelastic fluid Couette flow in an irregular channelwith varying mass diffusion. They have pointed out the effect of dusty viscoelastic fluid on flow, heat and mass transfer in an irregular channel. Investigating the references regarding the existence of multiple solutions of the governing equation the following are of great works. Turkyilmazoglu[15] studied multiple solutions of heat and mass transfer of MHD slip flow for the viscoelasticfluid over a stretching sheet. Turkyilmazoglu[16] also discussed Dual and triple solutions for MHD slip flow of non-Newtonianfluid over a shrinking surface. Multiple analytic solutions of heat and mass transfer of MHD slip flow for two types of viscoelastic fluids over a stretching surface have been investigated by Turkyilmazoglu[17]. Moreover, Elbashbeshy and Ibrahim[18] investigated the effect of steady free convection flow with variable viscosity and thermal diffusivity along a vertical plate. Kafoussias and Williams[19] studied the thermal-diffusion and diffusion-thermo effects on the mixed free-forced convective and mass transfer steady laminar boundary layer flow over a vertical plate, with temperature dependent viscosity. Sajid and Hayat[20] investigated the radiation effects on the mixed convection flow over an exponentially stretching sheet and solved the problem analytically using homotopy analysis method. The numerical solution for the same problem was then given by Bidin and Nazar[21]. Recently, Poornima and Bhaskar Reddy[22] presented an analysis of the radiation effects on MHD free convective boundary layer flow of nanofluids over a nonlinear stretchingsheet. However, the interaction of radiation with mass transfer due to a stretching sheet has received little attention. Abolbashari *et al*.[23] studied entropy analysis for an unsteady MHD flow past a stretching permeable surface in nanofluid. Rashidi and Erfani[24] applied an analytical method for solving steady MHD convective and slip flow due to a rotating disk with viscous dissipation and Ohmic heating. Mixed convective heat transfer for MHD viscoelastic fluid flow over a porous wedge with thermal radiation is studied by Rashidi *et al.*[25]. Further, Rashidi *et al.*[26] studied an analytic approximate solution for MHD boundary layer viscoelastic fluid flow over continuously moving stretching surface by HAM with two auxiliary parameters. It has been experimentally verified that the diffusion of energy is caused by a composition gradient. This fact is known as the Dufour effect or the diffusion-thermo effect. The diffusion of species by a temperature gradient is termed as the Soret effect or the thermal diffusion effect. In most of the studies dealing with heat and mass transfer, these effects are neglected under the assumption that these are of smaller orders of magnitude described by Fourier's and Fick's laws. Anwar Be´g *et al.*[27] obtained the numerical solutions for the free convective flow induced by a stretching surface in the presence of Dufour and Soret effects. Tsai and Huang[28] analyzed Dufour and Soret effects on the Hiemenz flow over a stretching surface immersed in a porous medium. Afify[29] studied Dufour and Soret effects in heat and mass transfer in the flow induced by a stretching surface. Mishra[30] analyzed the effect of free convection and mass transfer on the flow of an elastico-viscous fluid (Walters B0 model) in a vertical channel. The diffusion thermo effect has been considered on the fully developed laminar flow with uniform plate temperature and concentration. Tripathy *et al.*[31] studied Chemical reaction effect on MHD free convective surface over a moving vertical plane through porous medium. Rashidi *et al.*[32] examined the heat and mass transfer for MHD viscoelastic fluid flow over a vertical stretching sheet with considering Soret and Dufour effects. Further, Baag *et al.*[33] discussed the numerical investigation on MHD micropolar fluid flow toward a stagnation point on a vertical surface with heat source and chemical reaction. Problems on fluid flow and mass transfer through porous media are the interest of not only mathematicians but also chemical engineers who have generally concerned with reacting and absorbing species, petroleum engineers, who have concerned with the miscible displacement process and civil engineers who are confronted with the problem of salt water encroachment of careful costal equiforce. Also, porous media are very widely used for heated body to maintain its temperature. To make the heat insulation of the surface more effective it is necessary to study the free convective effects under flow through porous media and to estimate effects on the heat transfer. Further, Gebhart and Pera[34] studied the laminar flows which arise in fluid due to the interaction of the gravity forces and density differences caused by the simultaneous diffusion of thermal energy and of chemical species neglecting the thermal diffusion and diffusion-thermo (Soret – Dufour) effects because the level of species concentration is very low. Sparrow *et al*.[35] analyzed the free convection flow with Soret–Dufour effects.

Recently, Krishna and M.G. Reddy[36] discussed MHD free convective rotating flow of visco-elastic fluid past an infinite vertical oscillating plate. Krishna and G.S. Reddy[37] discussed unsteady MHD convective flow of second grade fluid through a porous medium in a Rotating parallel plate channel with temperature dependent source. Krishna and Swarnalathamma[38] discussed the peristaltic MHD flow of an incompressible and electrically conducting Williamson fluid in a symmetric planar channel with heat and mass transfer under the effect of inclined magnetic field. Viscous dissipation and Joule heating are also taken into consideration. Swarnalathamma and Krishna[39] discussed the theoretical and computational study of peristaltic hemodynamic flow of couple stress fluids through a porous medium under the influence of magnetic field with wall slip condition. D. V. Krishna *et al*.[40] discussed the unsteady hydromagnetic flow of an incompressible viscous fluid in a rotating parallel plate channel with porous lining under the influence of uniform transverse magnetic field normal to the channel and it was extended by Krishna *et al.* [41]. Krishna *et al*.[42] studied the steady hydro magnetic flow of a couple stress fluid through a composite medium in a rotating parallel plate channel with porous bed on the lower half subjected to subjected to normal to the channel and extended the problem taking hall current into account by Krishna *et al.*[43]. Krishna and Malashetty[44] discussed the unsteady flow of an in compressible electrically conducting second grade fluid in rigidly rotating parallel plate channel bounded below by a sparsely packed porous bed subjected to normal to the channel and extended the problem taking hall current into account by Krishna and Malashetty[45].

In this paper, we have considered radiation effects on MHD flow of visco-elastic fluid past an impulsively started vertical porous plate with variable heat and mass transfer. The results are shown with the help of tables and graphs.

2. Formulation and Solution of the Problem:

We consider the unsteady MHD flow of visco-elastic fluid past an impulsively started vertical porous plate. The *x*- axis is taken along the plate in the upward direction and *y*-axis is taken normal to the plate. Initially the fluid and plate are at the same temperature. A transverse magnetic field B_0 of uniform strength is applied normal to the plate as shown in Figure 1. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially, the fluid and plate are at the same temperature T_1 and concentration C_1 in the stationary condition. At time $t > 0$, the plate is moving with a velocity $u = u_0$ in its own plane and the temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time.

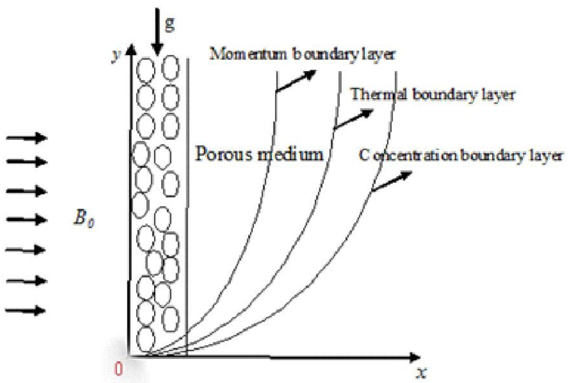

Figure 1: Physical configuration of the problem

The unsteady hydro magnetic equations of the MHD flow through porous medium are as:

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 u}{\partial y^2 \partial t} - \frac{\sigma_e B_0^2}{\rho}u - \frac{\nu}{K}u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C}{\partial t} = D_1 \frac{\partial^2 C}{\partial y^2} \tag{3}$$

The initial and boundary conditions

$$u = 0, T = T_\infty, C = C_\infty, \quad t \le 0, \ \forall \ y \tag{4}$$

$$u = u_0, T = T_\infty + (T_w - T_\infty)At, C = C_\infty + (C_w - C_\infty)At, \quad y = 0 \tag{5}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, \quad y \to \infty \tag{6}$$

Where $A = \frac{u_0^2}{\nu}$, The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^4 - T^4) \tag{7}$$

Considering the temperature difference within the flow sufficiently small, T^4 can be expressed as the linear function of temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

Using equations (7) and (8), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} + 16a^*\sigma(T_\infty^3 - T^4) \tag{9}$$

Introducing the following non- dimensional quantities:

$$u^* = \frac{u}{u_0}, y^* = \frac{yu_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}, \mu = \rho\nu, t^* = \frac{tu_0^2}{\nu}$$

Making use of non-dimensional variables, the equations (1), (2) and (9) leads to (dropping asterisks)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial^3 u}{\partial y^2 \partial t} - \left(M^2 + \frac{1}{D}\right)u + \mathrm{Gr}\,\theta + \mathrm{Gm}\,C \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\mathrm{Pr}}\theta \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \tag{12}$$

With boundary conditions

$$u = 0, \theta = 0, C = 0, \quad t \le 0, \ \forall \ y \tag{13}$$

$$u = 1, \theta = t, C = t, \quad y = 0 \tag{14}$$

$$u \to 0, \theta \to 0, C \to 0, \ as \ y \to \infty \tag{15}$$

Where, $R = \frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2}$ is the Radiation parameter, $M^2 = \frac{\sigma_e B_0^2 \nu}{\rho u_0^2}$ is the Hartmann number, $D = \frac{Ku_0^2}{\nu^2}$ is the Darcy parameter, $\mathrm{Gr} = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}$ is the thermal Grashof number, $\mathrm{Gm} = \frac{g\beta^*\nu(C_w - C_\infty)}{u_0^3}$ the mass Grashof number, $\mathrm{Pr} = \frac{\mu C_p}{k}$ is Prandtl parameter, $\alpha = \frac{\alpha_1 u_0^2}{\rho\nu^2}$ is the visco-elastic parameter and $Sc = \frac{\nu}{D_1}$ is the Schmidt number.

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13) to (15), are solved by the usual Laplace transform technique. With help of Hetnarski's (1975) development has also been taken. The solutions derived are given below. Transforming equation (12) we get,

$$s\bar{C}(y,s)-C(y,0)=\frac{1}{Sc}\frac{d^2\bar{C}}{dy^2} \tag{16}$$

Using boundary conditions (13) to (15), we have,

$$\frac{d^2\bar{C}}{dy^2}-s\,Sc\bar{C}(y,s)=0 \tag{17}$$

The solution of the equation (16) is

$$\bar{C}(y,s)=Ae^{\sqrt{sSc}y}+Be^{-\sqrt{sSc}y} \tag{18}$$

Where A and B are arbitrary constants.

Again using above boundary conditions (13) and (14), we get,

$$\bar{C}(y,s)=\frac{1}{s^2}e^{-\sqrt{sSc}y} \tag{19}$$

Taking inverse Laplace transform for the equation (19),

$$C(y,t)=t\left(1+2\left(\frac{y}{2\sqrt{t}}\right)^2 Sc\right)erfc\left(\left(\frac{y}{2\sqrt{t}}\right)\sqrt{Sc}\right)-\frac{2\left(\frac{y}{2\sqrt{t}}\right)\sqrt{Sc}}{\sqrt{\pi}}e^{-\left(\frac{y}{2\sqrt{t}}\right)^2 Sc} \tag{20}$$

Also transforming equation (11);

$$s\bar{\theta}(y,s)-\theta(y,0)=\frac{1}{\Pr}\frac{d^2\bar{\theta}}{dy^2}-\frac{R}{\Pr}\bar{\theta}(y,s) \tag{21}$$

Using boundary conditions (13) and (14), it reduces to:

$$\frac{d^2\bar{\theta}}{dy^2}-(R+s\Pr)\bar{\theta}(y,s)=0 \tag{22}$$

The solution of this equation (21)

$$\bar{\theta}(y,s)=Ce^{y\sqrt{R+s\Pr}}+Ee^{-y\sqrt{R+s\Pr}} \tag{23}$$

Where, C and E are arbitrary constants. Values of C and E can be computed using (13) and (14), we obtain

$$\bar{\theta}(y,s)=\frac{1}{s^2}e^{-y\sqrt{\Pr\left(s+\frac{R}{\Pr}\right)}} \tag{24}$$

Taking inverse Laplace transform for the equation (23), we obtain

$$\theta(y,t)=\frac{t}{2}\left(a_1e^{2\xi\sqrt{Rt}}erfc(\xi\sqrt{\Pr}+\sqrt{ct})+a_2e^{-2\xi\sqrt{Rt}}erfc(\xi\sqrt{\Pr}-\sqrt{ct})\right) \tag{25}$$

Similarly, again taking the Laplace transform to the equation (10) and making use of the initial and boundary conditions (13) to (15), it reduces to

$$(1+s\alpha)\frac{d^2\bar{u}}{dy^2}-\left[s+\left(M^2+\frac{1}{D}\right)\right]\bar{u}(y,s)=-\mathrm{Gr}L\{\theta(y,t)\}-\mathrm{Gm}L\{C(y,t)\} \tag{26}$$

The solution of the equation (26) is

$$\bar{u}(y,s)=Fe^{y\sqrt{\frac{s+\left(M^2+\frac{1}{D}\right)}{1+s\alpha}}}+Ge^{-y\sqrt{\frac{s+\left(M^2+\frac{1}{D}\right)}{1+s\alpha}}}+\frac{\mathrm{Gr}}{(1-\Pr)(1+s\alpha)}\frac{e^{-y\sqrt{\Pr}\sqrt{s+\frac{R}{\Pr}}}}{s^2(s-a_3)}+\frac{\mathrm{Gm}}{(1-Sc)(1+s\alpha)}\frac{e^{-y\sqrt{sSc}}}{s^2(s-a_4)} \tag{27}$$

Applying the boundary conditions (13) and (14) for (26), we obtain

$$\bar{u}(y,s)=\frac{1}{s}e^{-y\sqrt{\frac{s+\left(M^2+\frac{1}{D}\right)}{1+s\alpha}}}+\frac{\mathrm{Gr}}{1-\Pr}\left(\frac{e^{-y\sqrt{\Pr}\sqrt{s+\frac{R}{Pr}}}-e^{-y\sqrt{\frac{s+\left(M^2+\frac{1}{D}\right)}{1+s\alpha}}}}{s^2(s-a_3)(1+s\alpha)}\right)+\frac{\mathrm{Gm}}{1-Sc}\left(\frac{e^{-y\sqrt{sSc}}-e^{-y\sqrt{\frac{s+\left(M^2+\frac{1}{D}\right)}{1+s\alpha}}}}{s^2(s-a_4)(1+s\alpha)}\right) \tag{28}$$

Taking the inverse Laplace transform to the equation (28), we obtain the velocity as

$$u(y,t)=a_5\,e^{-y\sqrt{\left(M^2+\frac{1}{D}\right)}}\,erfc\left(\xi-\sqrt{\left(M^2+\frac{1}{D}\right)t}\right)+a_6\,e^{y\sqrt{\left(M^2+\frac{1}{D}\right)}}\,erfc\left(\xi+\sqrt{\left(M^2+\frac{1}{D}\right)t}\right)+$$

$$-\left[e^{-y\sqrt{\Pr(a_3+(R/\Pr))}}\,erfc(\xi\sqrt{\Pr}-\sqrt{(a_3+(R/\Pr))t})+e^{y\sqrt{\Pr(a_3+(R/\Pr))}}\,erfc(\xi\sqrt{\Pr}+\sqrt{(a_3+(R/\Pr))t})\right]\frac{a_{11}}{2}e^{a_3t}$$

$$-\left[e^{-y\sqrt{\Pr(-(1/\alpha)+(R/\Pr))}}\,erfc(\xi\sqrt{Pr}-\sqrt{(-(1/\alpha)+(R/\Pr))t})+\right.$$

$$\left.e^{y\sqrt{\Pr(-(1/\alpha)+(R/\Pr))}}\,erfc(\xi\sqrt{\Pr}+\sqrt{(-(1/\alpha)+(R/\Pr))t})\right]\frac{a_{11}}{2}e^{-(1/\alpha)t}$$

$$-\left(a_7\,e^{-y\sqrt{\Pr(R/\Pr)}}\,erfc(\xi\sqrt{\Pr}-\sqrt{(R/\Pr)t})+a_8\,e^{y\sqrt{\Pr(R/\Pr)}}\,erfc(\xi\sqrt{\Pr}+\sqrt{(R/\Pr)t})\right)$$

$$-\left[e^{-y\sqrt{\Pr\left(M^2+\frac{1}{D}+a_3\right)}}\,erfc\left(\xi\sqrt{\Pr}-\sqrt{\Pr\left(M^2+\frac{1}{D}+a_3\right)t}\right)+\right.$$

$$\left.e^{y\sqrt{\Pr\left(M^2+\frac{1}{D}+a_3\right)}}\,erfc\left(\xi\sqrt{\Pr}+\sqrt{\Pr\left(M^2+\frac{1}{D}+a_3\right)t}\right)\right]\frac{a_{11}}{2}e^{a_3t}$$

$$-\left[e^{-y\sqrt{\Pr\left(\frac{M^2+\frac{1}{D}+a_3}{1+a_3\alpha}\right)}}\,erfc\left(\xi\sqrt{\Pr}-\sqrt{\Pr\left(\frac{M^2+\frac{1}{D}+a_3}{1+a_3\alpha}\right)t}\right)+\right.$$

$$\left.e^{y\sqrt{\Pr\left(\frac{M^2+\frac{1}{D}+a_3}{1+a_3\alpha}\right)}}\,erfc\left(\xi\sqrt{\Pr}+\sqrt{\Pr\left(\frac{M^2+\frac{1}{D}+a_3}{1+a_3\alpha}\right)t}\right)\right]\frac{a_{11}}{2}e^{-(1/\alpha)t}$$

$$+\left[e^{-y\sqrt{a_4Sc}}\,erfc(\xi\sqrt{Sc}-\sqrt{a_4t})+e^{y\sqrt{a_4Sc}}\,erfc(\xi\sqrt{Sc}+\sqrt{a_4t})\right]\frac{a_{12}}{2}e^{a_4t}$$

$$+\left[e^{-y\sqrt{-(1/\alpha)Sc}}\,erfc(\xi\sqrt{Sc}-\sqrt{-(1/\alpha)t})+e^{y\sqrt{a_4Sc}}\,erfc(\xi\sqrt{Sc}+\sqrt{-(1/\alpha)t})\right]\frac{a_{12}}{2}e^{-(1/\alpha)t}$$

$$-\left[e^{-y\sqrt{\left(\frac{M^2+\frac{1}{D}+a_4}{1+a_4\alpha}\right)}}\,erfc\left(\xi-\sqrt{\left(\frac{M^2+\frac{1}{D}+a_4}{1+a_4\alpha}\right)t}\right)+e^{y\sqrt{\left(\frac{M^2+\frac{1}{D}+a_4}{1+a_4\alpha}\right)}}\,erfc\left(\xi+\sqrt{\left(\frac{M^2+\frac{1}{D}+a_4}{1+a_4\alpha}\right)t}\right)\right]\frac{a_{12}}{2}e^{a_4t}$$

$$-\left[e^{-y\sqrt{M^2+\frac{1}{D}-\frac{1}{\alpha}}}\,erfc\left(\xi-\sqrt{\left(M^2+\frac{1}{D}-\frac{1}{\alpha}\right)t}\right)+e^{y\sqrt{\left(M^2+\frac{1}{D}-\frac{1}{\alpha}\right)}}\,erfc\left(\xi+\sqrt{M^2+\frac{1}{D}-\frac{1}{\alpha}t}\right)\right]\frac{a_{12}}{2}e^{-(1/\alpha)t}$$

$$-a_{12}\left[1+a_4t(1+2\xi^2Sc)erfc(\xi\sqrt{Sc})+\frac{2a_4t\xi\sqrt{Sc}}{\sqrt{\pi}}e^{-\xi^2Sc}\right] \tag{29}$$

The non-dimensional shear stress is given by

$$\tau = -\left(\frac{du}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{du}{d\xi}\right)_{\xi=0} \tag{30}$$

The non-dimensional Nusselt number is given by

$$Nu = -\left(\frac{d\theta}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{d\theta}{d\xi}\right)_{\xi=0} \tag{31}$$

The non-dimensional Sherwood number is given by

$$Sh = -\left(\frac{dC}{dy}\right)_{y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{dC}{d\xi}\right)_{\xi=0} \tag{32}$$

3. Results and Discussion

We have discussed the exact analysis and are presented to investigate the combined effects of heat and mass transfer on the MHD flow of visco-elastic fluid bounded by loosely packed porous medium in an impulsively started vertical plate with variable heat and mass transfer. The expressions for the velocity, temperature and concentration are obtained by using Laplace transform technique and also discussed the physical behaviour of the dimensionless parameters such as Hartmann number *M*, Darcy parameter *D* (Permeability parameter), Radiation parameter *R*, α visco-elastic parameter, thermal Grashoff number Gr, mass Grashoff number Gm, Prandtl number Pr and Schmidt number *Sc*. Figures (2-13) have been displayed for the velocity, temperature and concentration. Skin friction, Nusselt number and Sherwood number are shown in Tables (1-3). The velocity, temperature and concentration profiles for some realistic values of Prandtl number Pr (Pr = 0.71, 0.16, 3 for the saturated liquid Freon at 273.3^{o} and Pr = 7 for water) and Schmidt number Sc (Sc = 0.2 for hydrogen) respectively. From figure (2), this presents the velocity profile for different values of *M* being other parameters fixed. We noticed that the velocity decreases with increasing the Hartmann number *M*. It is due to the fact that the application of transverse magnetic field results a resistive type force (Lorentz force) similar to drag force and upon increasing the intensity of the magnetic field which leads to the deceleration of the flow. Figure (3) is sketched in order to explore the variations of permeability parameter *D*. It is found that the magnitude of the velocity increases with increasing the values of permeability parameter *D*. This is due to the fact that increasing the permeability reduces the drag force which assists the fluid considerably to move fast. Likewise the magnitude of the velocity *u* reduced continuously with increasing the radiation parameter R from figure (4). The magnitude of the velocity enhances with increasing visco-elastic parameter α (Fig. 5). The variation of velocity for different values of dimensionless time *t* and Prandtl number Pr is shown in figure (6 and 7). It is noticed that velocity increases with increasing time *t*. It is also observed from the figure (7) that the magnitude of the velocity *u* decreases with increasing Prandtl number Pr. It is clear from figure (8), the velocity decreases with increasing the thermal Grashof number Gr (cooling plate), where as there sharp enhancement in velocity for heating the plate, this is increase sustains away from the plate. Figure (9) reveals that the magnitude of the velocity increases with increasing mass Grashoff number Gm throughout the fluid region. Similarly the same phenomenon is observed with increasing Schmidt number *Sc* from figure (10). The effect of radiation parameter *R* on the temperature profile is shown in figure (11). It is found that the temperatures, being as decreasing function of *R*, decelerates the fluid flow and reduce the fluid velocity. Such an effect may also be expected, here as increasing radiation parameter *R* makes the fluid thick and ultimately causes the temperature and thermal boundary layer thickness to reduce. Hence it is observed that the temperature decreases with increasing the radiation parameter *R* throughout the fluid region. The Prandtl number actually describes the relationship between momentum diffusivity and thermal diffusivity and hence control the relative thickness of the momentum and thermal boundary layers. From figure (12), we observed that the temperature reduces with increasing the values of Prandtl number Pr, it is also observed that the thermal boundary layer thickness is maximum near the plate and reduces with increasing distances from leading edge and finally approaches to zero. It is also justified due to the fact that thermal conductivity of the fluid decreases with increasing Prandtl number Pr and hence decreases the thermal boundary layer and the temperature profile. Figure (13) depicts the increasing values of Schmidt number *Sc* lead to fall the concentration profiles throughout the fluid.

The numerical values of the skin friction (τ), Nusselt number (*Nu*) and Sherwood number (*Sh*) are computed and are tabulated in the tables (1-3), in all these tables the comparison of each parameter is made with first row in the corresponding table. It found from table (1), the effect of each parameter on the skin friction shows that, τ enhances with increasing *R*, *D*, α, Pr, Gr, Gm, *Sc* and time *t*, while decreases with *M* and –Gr. It is observed from table (2) that Nusselt number *Nu* increases with increasing *R*, Pr and *t*. From table (3) we observed that Sherwood number goes on increasing with increasing *Sc* and *t*.

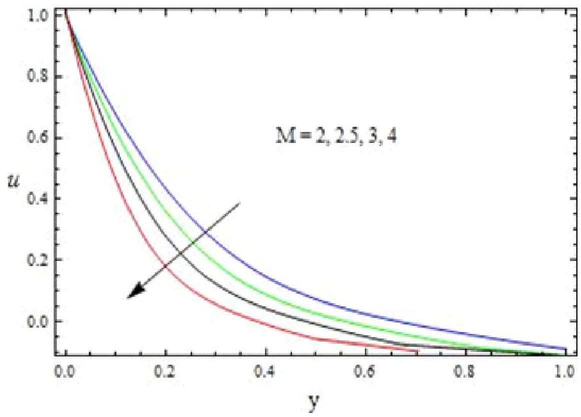

Fig 2. The velocity Profile for u against M with $\alpha = 1$; D=1; P= 0.71; t=0.1; Sc=2; R=1; Gr=5; Gm=10

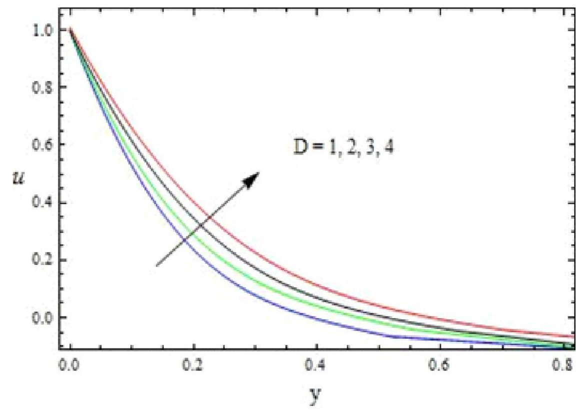

Fig 3. The velocity Profile for u against D with $\alpha = 1$; M=2; P= 0.71; t=0.1; Sc=2; R=1; Gr=5; Gm=10

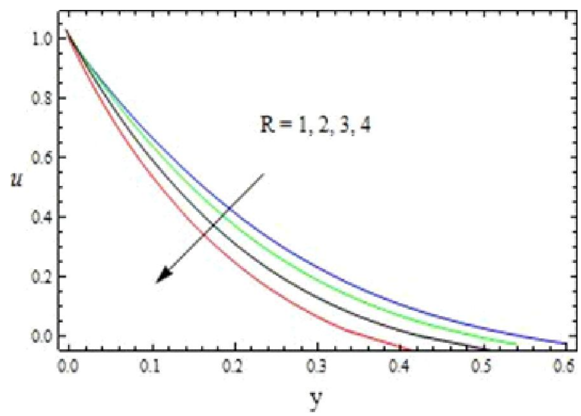

Fig 4. The velocity Profile for u against R with $\alpha = 1$; D=1; P= 0.71; t=0.1; Sc=2; M=2; Gr=5; Gm=10

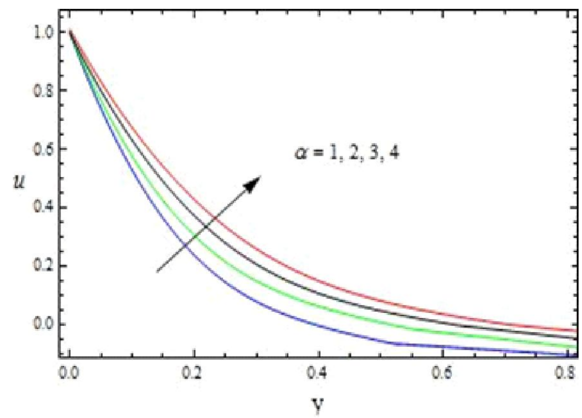

Fig 5. The velocity Profile for u against α with D=1; P= 0.71; t=0.1; R=1; Sc=2; M=2; Gr=5; Gm=10

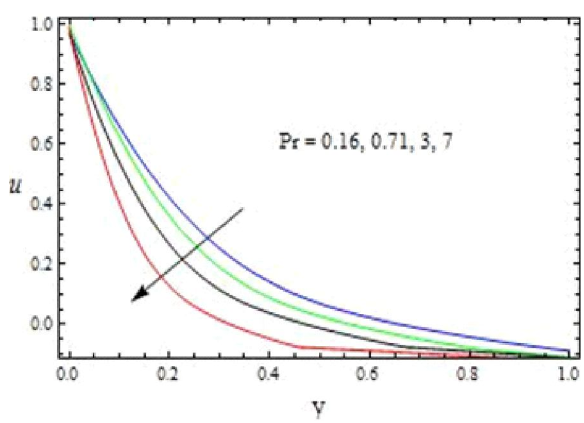

Fig 6. The velocity Profile for u against Pr and t with $\alpha = 1$; M=2; D=1; t=0.1; Sc=2; R=1; Gr=5; Gm=10

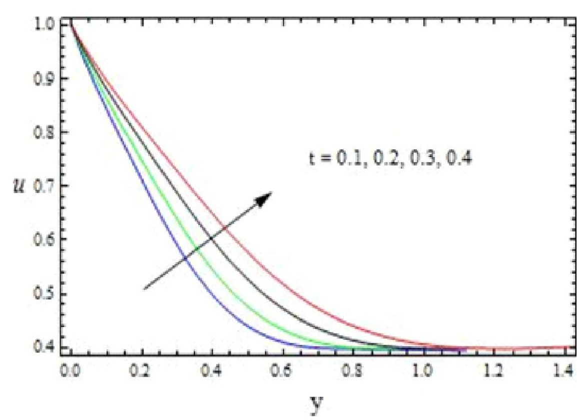

Fig 7. The velocity Profile for u against t with $\alpha = 1$; M=2; D=1; t=0.1; Sc=2; R=1; Gr=5; Gm=10

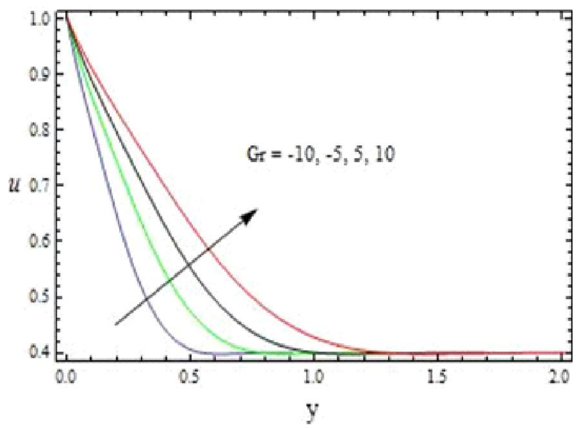

Fig 8. The velocity Profile for u against Gr with $\alpha = 1$; M=2; D=1; P=0.71, ; Sc=2; R=1; t=0.1; Gm=10

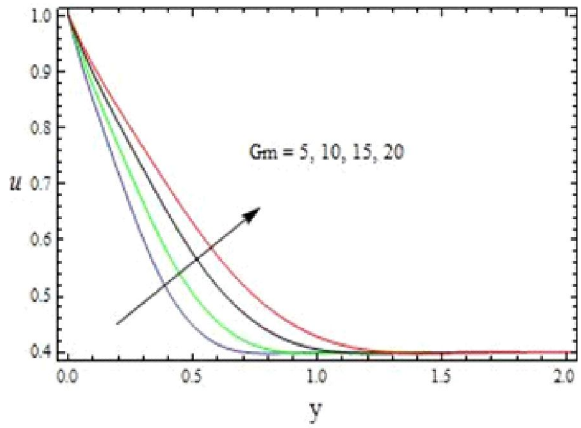

Fig 9. The velocity Profile for u against Gm with $\alpha = 1$; M=2; D=1; P=0.71, Sc=2; R=1; t=0.1; Gr=5

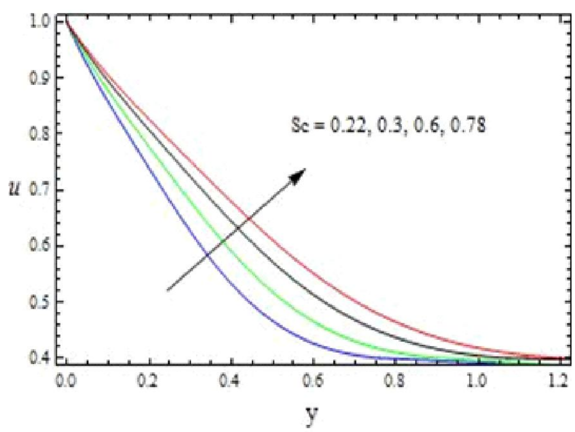

Fig 10. The velocity Profile for u against Sc with $\alpha = 1$; M=2; D=1; P=0.71, R=1; t=0.1; Gr=5; Gm=10

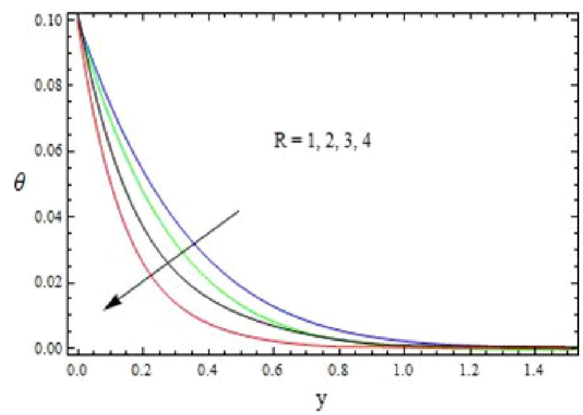

Fig 11. The Temperature Profile for θ against R with P=0.71; t=0.1

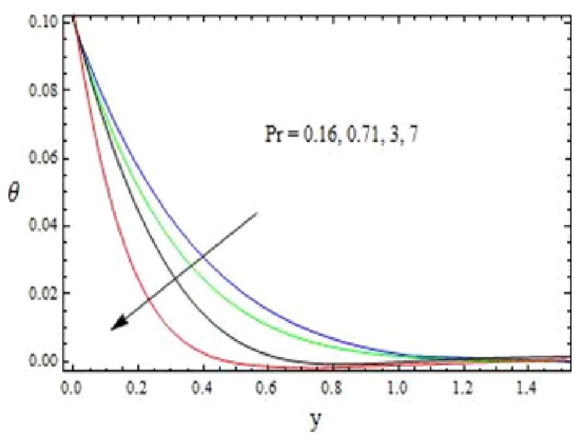

Fig 12. The Temperature Profile for θ against Pr with R=2; t=0.1

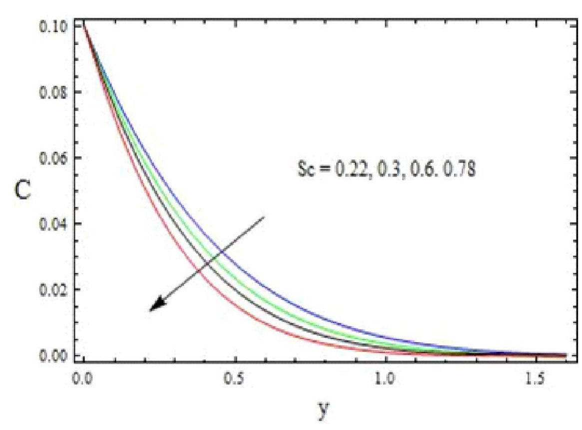

Fig 13. The Concentration Profile for C against Sc with t=0.1

Table 1. The effects of various parameters on Skin friction (shear stress (τ))

R	M	K	α	Pr	Gr	Gm	Sc	t	τ
1	2	1	1	0.71	5	10	2	0.2	3.64785
2	2	1	1	0.71	5	10	2	0.2	4.29045
3	2	1	1	0.71	5	10	2	0.2	4.85887
1	3	1	1	0.71	5	10	2	0.2	3.47988
1	4	1	1	0.71	5	10	2	0.2	2.14955
1	2	2	1	0.71	5	10	2	0.2	3.71004
1	2	3	1	0.71	5	10	2	0.2	3.71477
1	2	1	2	0.71	5	10	2	0.2	3.85689
1	2	1	3	0.71	5	10	2	0.2	4.12658
1	2	1	1	0.16	5	10	2	0.2	3.14458
1	2	1	1	3	5	10	2	0.2	5.22985
1	2	1	1	7	5	10	2	0.2	10.4478
1	2	1	1	0.71	10	10	2	0.2	3.92478
1	2	1	1	0.71	15	10	2	0.2	4.17898
1	2	1	1	0.71	-10	10	2	0.2	2.91145
1	2	1	1	0.71	-15	10	2	0.2	2.65897
1	2	1	1	0.71	5	5	2	0.2	1.96698
1	2	1	1	0.71	5	15	2	0.2	5.37855
1	2	1	1	0.71	5	20	2	0.2	7.08801
1	2	1	1	0.71	5	10	3	0.2	3.78695
1	2	1	1	0.71	5	10	4	0.2	4.36989
1	2	1	1	0.71	5	10	5	0.2	4.99258
1	2	1	1	0.71	5	10	2	0.3	4.84685
1	2	1	1	0.71	5	10	2	0.4	6.06858
1	2	1	1	0.71	5	10	2	0.5	7.04522

Table 2. The effects of various parameters on the Rate of heat transfer (Nu)

R	Pr	t	Nu
1	0.71	0.1	0.195870
2	0.71	0.1	0.216376
3	0.71	0.1	0.235839
4	0.71	0.1	0.254358
1	0.16	0.1	0.107555
1	3	0.1	0.634710
1	7	0.1	1.393160
1	0.71	0.2	0.331442
1	0.71	0.3	0.461249
1	0.71	0.4	0.588593

Table 3. The effects of various parameters on the Sherwood number (Sh)

Sc	t	Sh
2	**0.1**	0.104512
3	0.1	0.226218
4	0.1	0.356825
5	0.1	0.493120
2	**0.2**	0.147802
2	**0.3**	0.181019
2	**0.4**	0.209023
2	**0.5**	0.233695

4. Conclusions

We have studied the unsteady MHD flow of visco-elastic fluid past an impulsively started vertical porous plate with variable heat and mass transfer. The conclusions are made as following.

1. The velocity decreases with increasing the intensity of the magnetic field.
2. The velocity increases with increasing the permeability parameter *D* or visco-elastic parameter α.
3. The magnitude of the velocity enhances and reduced continuously with increasing the radiation parameter R.
4. The velocity increases with increasing time *t*. It is also observed that the magnitude of the velocity *u* decreases with increasing Prandtl number Pr.
5. The velocity decreases with increasing the thermal Grashof number Gr (cooling plate), whereas there sharp enhancement in velocity for heating the plate, this is increase sustains away from the plate.
6. The magnitude of the velocity increases with increasing mass Grashof number Gm throughout the fluid region. The same phenomenon is observed with increasing Schmidt number *Sc*.
7. The temperature decreases with increasing the radiation parameter R or Pr.
8. The increasing values of Schmidt number Sc lead to fall the concentration profiles throughout the fluid.
9. The skin friction enhances with increasing R, D, α, Pr, Gr, Gm, *Sc* and time *t*, while decreases with M and –Gr.
10. Nusselt number Nu increases with increasing R, Pr and *t*.
11. Sherwood number goes on increasing with increasing *Sc* and *t*. Nusselt number Nu increases with increasing R, Pr and *t*.
12. Sherwood number goes on increasing with increasing *Sc* and *t*.

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Appendix:

$$\xi=\frac{y}{2\sqrt{t}},\ a_1=1+\frac{\xi\,\mathrm{Pr}}{\sqrt{Rt}},\ a_2=1-\frac{\xi\,\mathrm{Pr}}{\sqrt{Rt}},\ a_3=\frac{R-\left(M^2+\frac{1}{D}\right)}{1-\mathrm{Pr}},\ a_4=\frac{\left(M^2+\frac{1}{D}\right)}{Sc-1}$$

$$a_5=\frac{1}{2}\left(a_9+a_{10}\left(t-\frac{y}{2\sqrt{M^2+(1/D)}}\right)\right),\ a_6=\frac{1}{2}\left(a_9+a_{10}\left(t+\frac{y}{2\sqrt{M^2+(1/D)}}\right)\right)$$

$$a_7=\frac{a_{11}}{2}\left(1+a_3t-\frac{ya_3\sqrt{\mathrm{Pr}}}{2\sqrt{R/\mathrm{Pr}}}\right),\quad a_8=\frac{a_{11}}{2}\left(1+a_3t+\frac{ya_3\sqrt{\mathrm{Pr}}}{2\sqrt{R/\mathrm{Pr}}}\right),\ a_9=1+a_{11}+a_{12},$$

$$a_{10}=\frac{\mathrm{Gr}}{a_3(1-\mathrm{Pr})}+\frac{\mathrm{Gm}}{a_4(1-Sc)},\ a_{11}=\frac{\mathrm{Gr}}{a_3^2(1-\mathrm{Pr})},\quad a_{12}=\frac{\mathrm{Gm}}{a_4^2(1-Sc)}$$